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Recognized Under 2(f) of UGC Act 1956,ISO 9001:2015 Certified
ESTD : 2001

Department of Civil Engineering

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Lecture Notes

B. Tech II YEAR – II SEM

Prepared by

G PUSHPALATHA
Assistant Professor
CED

CE403PC: STRENGTH OF MATERIALS – II

Pre-Requisites: Strength of Materials - I

Course Objectives: The objective of this Course is

- To understand the nature of stresses developed in simple geometries shafts, springs, columns &cylindrical and spherical shells for various types of simple loads.
- To calculate the stability and elastic deformation occurring in various simple geometries for different types of loading.
- To understand the unsymmetrical bending and shear center importance for equilibrium conditions in a structural member of having different axis of symmetry.

Course Outcome: On completion of the course, the student will be able to:

- Describe the concepts and principles, understand the theory of elasticity, and perform calculations, relative to the strength of structures and mechanical components in particular to torsion and direct compression.
- To evaluate the strains and deformation that will result due to the elastic stresses developed within the materials for simple types of loading.
- Analyze strength and stability of structural members subjected to Direct, and Direct and Bending stresses.
- Understand and evaluate the shear center and unsymmetrical bending.

UNIT – I

Torsion of Circular Shafts: Theory of pure torsion – Derivation of Torsion equation -Assumptions made in the theory of pure torsion – Polar section modulus – Power transmitted by shafts – Combined bending and torsion – Design of shafts according to theories of failure.

Springs: Introduction – Types of springs – deflection of close and open coiled helical springs under axial pull and axial couple – springs in series and parallel.

UNIT – II

Columns and Struts: Introduction – Types of columns – Short, medium and long columns – Axially loaded compression members – Crushing load – Euler's theorem for long columns- assumptions- derivation of Euler's critical load formulae for various end conditions – Equivalent length of a column – slenderness ratio – Euler's critical stress – Limitations of Euler's theory– Long columns subjected to eccentric loading – Secant formula – Empirical formulae — Rankine – Gordon formula- Straight line formula – Prof. Perry's formula.

BEAM COLUMNS: Laterally loaded struts – subjected to uniformly distributed and concentrated loads.

UNIT - III

Direct and Bending Stresses: Stresses under the combined action of direct loading and bending moment, core of a section – determination of stresses in the case of retaining walls, chimneys and dams – conditions for stability-Overturning and sliding – stresses due to direct loading and bending moment about both axis.

UNIT – IV

Thin Cylinders: Thin seamless cylindrical shells – Derivation of formula for longitudinal and circumferential stresses – hoop, longitudinal and Volumetric strains – changes in diameter, and volume of thin cylinders – Thin spherical shells.

Thick Cylinders: Introduction - Lame's theory for thick cylinders – Derivation of Lame's formulae – distribution of hoop and radial stresses across thickness – design of thick cylinders – compound cylinders – Necessary difference of radii for shrinkage.

UNIT – V

Unsymmetrical Bending:

Introduction – Centroidal principal axes of section –Moments of inertia referred to any set of rectangular axes – Stresses in beams subjected to unsymmetrical bending – Principal axes – Resolution of bending moment into two rectangular axes through the centroid – Location of neutral axis.

Shear Centre: Introduction - Shear center for symmetrical and unsymmetrical (channel, I, T and L) sections.

TEXT BOOKS:

1. Strength of Materials by R.K Rajput, S. Chand & Company Ltd.
2. Mechanics of Materials by Dr. B. C Punmia, Dr. Ashok Kumar Jain and Dr. Arun Kumar Jain
3. Strength of Materials by R. Subramanian, Oxford University Press.

REFERENCE BOOKS:

1. Mechanics of Materials by R.C. Hibbeler, Pearson Education
2. Engineering Mechanics of Solids by Popov E.P. Prentice-Hall Ltd
3. Strength of Materials by T.D.Gunneswara Rao and M.Andal, Cambridge Publishers
4. Strength of Materials by R. K. Bansal, Lakshmi Publications House Pvt. Ltd.
5. Fundamentals of Solid Mechanics by M. L. Gambhir, PHI Learning Pvt. Ltd

UNIT-1

TORSION OF CIRCULAR SHAFTS

Simple or Single shaft

T ζ Gθ
--- = ---- = ------- is general equation to stress and twist due to torsion.
Ip r L

T = Torque or Torsion or Angular Velocity obtained from power Ip = Polar moment of inertia is sum of I xxand I yy ζ
= Shear stress in shaft r = radius of shaft
L = Length of shaft θ=Angle of twist in radian. G or C= Modulus of rigidity

Convert to radian 180

P =Hollow shaft Π (D
Ip = ----------------- Ip = I/2 only for circular section 32
D - External dia and d – internal dia

Solid shaft d = 0
Therefore, Π DIp =Strength of shaft

Angle of twist is,

Torsional rigidity is the product of G and Ip which is GIp. Zp is known as polar modulus which is ratio of Polar inertia over the distance from NA.

Conditions: Torque is same in shafts T 1= T 2
Twist θ = θ 1+ θ 2Shafts rotate in same direction Twist θ = θ 1- θ 2Shafts rotate in opposite direction
Choose the least Torque between shafts for safe stress and angle of twist.

Shafts in parallel:

Conditions: Total Torque T = T 1+ T 2

Twist is same in both shaft θ 1= θ 2

The shafts may be of same material or different material, which is known as composite shaft.

Strain energy or Torsional resilience in shaft:

It is the amount of energy stored when the shaft is in twisted position.

Torsional energy U = Average Torque x angle of twist

T x θ

When U is divided by the volume of the shaft, is known as strain energy per unit volume.

Shaft coupled:

The shaft is joined together when the length is not sufficient this is known as coupling of shaft. It is done in two methods. 1. Using bolts

2. Using key

Bolt method

T can be obtained from shaft expression for bolt and keyed shaft. ζIp------2πNT T = ----

-- or from Power expression P = ---------------------------r

60

T is torque in shaft which is transmitted to the coupled shaft through bolts or key.

Therefore torque in bolts or key is equal to torque in shaft.

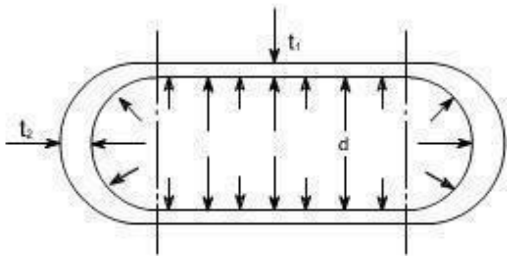
T = no. of bolts **x** area of bolt **x** stress in bolt **x** radius of bolt circle **Therefore T = n x Π**

d b

Cylindrical Vessel with Hemispherical Ends:

Let us now consider the vessel with hemispherical ends. The wall thickness of the cylindrical and hemispherical portion is different. While the internal diameter of both the portions is assumed to be equal

Let the cylindrical vassal is subjected to an internal pressure p.



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For the Cylindrical Portion

hoop or circumferential stress $= \sigma_{HC}$ 'c' here synifies the cylindrical portion.

$$= \frac{pd}{2t_1}$$

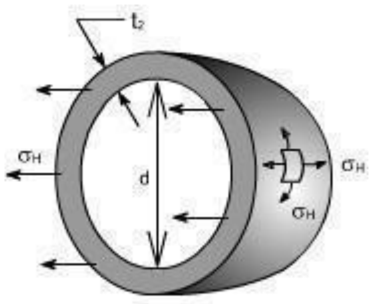
longitudnal stress $= \sigma_{LC}$

$$= \frac{pd}{4t_1}$$

hoop or circumferential strain $\in_2 = \frac{\sigma_{HC}}{E} - \nu\frac{\sigma_{LC}}{E} = \frac{pd}{4t_1E}[2-\nu]$

$$\text{or} \quad \boxed{\in_2 = \frac{pd}{4t_1E}[2-\nu]}$$

For The Hemispherical Ends:



Because of the symmetry of the sphere the stresses set up owing to internal pressure will be two mutually perpendicular hoops or circumferential stresses of equal values. Again the radial stresses are neglected in comparison to the hoop stresses as with this cylinder having thickness to diametre less than1:20.

Consider the equilibrium of the half – sphere

Force on half-sphere owing to internal pressure = pressure x projected Area

$= p. < d^2/4$

Resisting force $= \sigma_H \, . \, \pi . d . t_2$

$\therefore \quad p.\frac{\pi . d^2}{4} = \sigma_H \, . \, \pi d . t_2$

$\Rightarrow \; \sigma_H \text{ (for sphere)} = \frac{pd}{4t_2}$

similarly the hoop strain $= \frac{1}{E}\left[\sigma_H - \nu . \sigma_H\right] = \frac{\sigma_H}{E}[1-\nu] = \frac{pd}{4t_2E}[1-\nu]$ or $\boxed{\epsilon_{2s} = \frac{\mathbf{pd}}{\mathbf{4t_2E}}[\mathbf{1-\nu}]}$

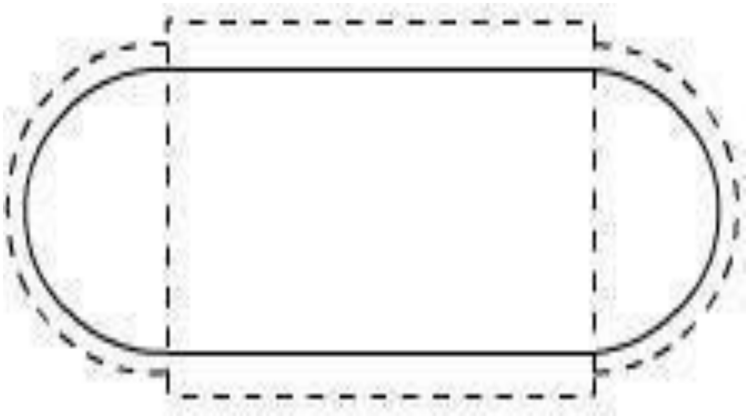


Fig – shown the (by way of dotted lines) the tendency, for the cylindrical portion and the spherical ends to expand by a different amount under the action of internal pressure. So owing to difference in stress, the two portions (i.e. cylindrical and spherical ends) expand by a different amount. This incompatibly of deformations causes a local bending and sheering stresses in the neighborhood of the joint. Since there must be physical continuity between the ends and the cylindrical portion, for this reason, properly curved ends must be used for pressure vessels.

Thus equating the two strains in order that there shall be no distortion of the junction

$$\frac{pd}{4t_1E}[2-\nu] = \frac{pd}{4t_2E}[1-\nu] \quad \text{or} \; \frac{t_2}{t_1} = \frac{1-\nu}{2-\nu}$$

But for general steel works $\nu = 0.3$, therefore, the thickness ratios becomes

$t_2 / t_1 = 0.7/1.7$

i.e. the thickness of the cylinder walls must be approximately 2.4 times that of the hemispheroid ends for no distortion of the junction to occur.

SUMMARY OF THE RESULTS : Let us summaries the derived results

(A) The stresses set up in the walls of a thin cylinder owing to an internal pressure p are :

(i) Circumferential or loop stress

$H = pd/2t$

(ii) Longitudinal or axial stress

$L = pd/4t$

Where d is the internal diametre and t is the wall thic kness of the cylinder. then

Longitudinal strain **1 / E [L− H]**

Hoop **H** : _ /] · 1 **]** stain **E [**

(B) Change of internal volume of cylinder under pressure

$$= \frac{pd}{4tE}[5 - 4v]V$$

(C) Fro thin spheres circumferential or loop stress

$$\sigma_H = \frac{pd}{4t}$$

Thin rotating ring or cylinder

Consider a thin ring or cylinder as shown in Fig below subjected to a radial internal pressure p caused by the centrifugal effect of its own mass when rotating. The centrifugal effect on a unit length of the circumference is

$p = m \omega^2 r$

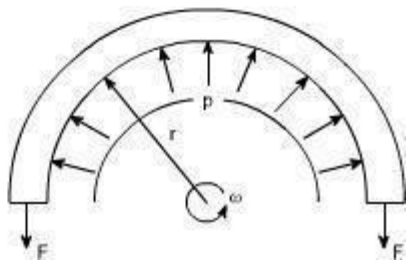


Fig 19.1: Thin ring rotating with constant angular velocity

Here the radial pressure „p' is acting per unit length and is caused by the centrifugal effect if its own mass when rotating.

Thus considering the equilibrium of half the ring shown in the figure, $2F = p \times 2r$
(assuming unit length), as 2r is the projected area $F = pr$

Where F is the hoop tension set up owing to rotation.

The cylinder wall is assumed to be so thin that the centrifugal effect can be assumed constant across the wall thickness.

F = mass x acceleration = $\mathbf{m \omega^2 r \times r}$

This tension is transmitted through the complete circumference and therefore is resisted by the complete cross – sectional area.

hoop stress = F/A = $\mathbf{m \omega^2 r^2 / A}$

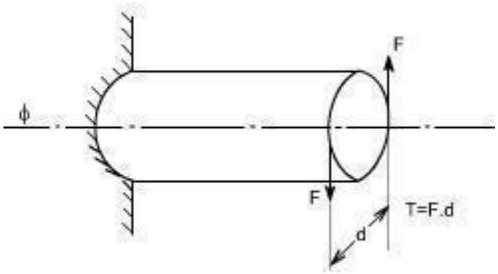
Where A is the cross – sectional area of the ring.

Now with unit length assumed m/A is the mass of the material per unit volume, i.e. the density < .

hoop stress $\sigma_H = \rho\ \omega^2\ \mathbf{r}^2$

Torsion of circular shafts

Definition of Torsion: Consider a shaft rigidly clamped at one end and twisted at the other end by a torque T = F.d applied in a plane perpendicular to the axis of the bar such a shaft is said to be in torsion.



Effects of Torsion: The effects of a torsional load applied to a bar are

(i) To impart an angular displacement of one end cross – section with respect to the other end.

(ii) To setup shear stresses on any cross section of the bar perpendicular to its axis.

Assumption:

(i) The materiel is homogenous i.e of uniform elastic properties exists throughout the material.

(ii) The material is elastic, follows Hook's law, with shear stress proportional to shear strain.

(iii) The stress does not exceed the elastic limit.

(iv) The circular section remains circular

(v) Cross section remain plane.

(vi) Cross section rotate as if rigid i.e. every diameter rotates through the same angle. Consider now the solid circular shaft of radius R subjected to a torque T at one end, the other end being fixed Under the action of this torque a radial line at the free end of the shaft twists through an of distortion of the shaft i.e the shear strain.

Since angle in rad ius = arc / Radius

arc AB = R

From the definition of Modulus of rigidity or Modulus of elasticity in shear

$$G = \frac{\text{shear stress}(\tau)}{\text{shear strain}(\gamma)}$$

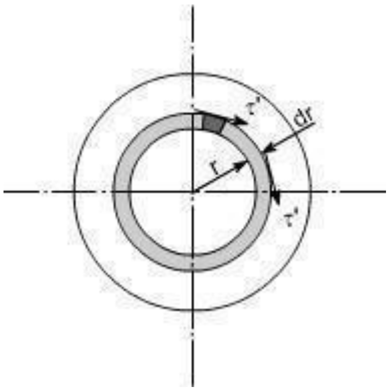
where γ is the shear stress set up at radius R.

Then $\frac{\tau}{G} = \gamma$

Equating the equations (1) and (2) we get $\frac{R\theta}{L} = \frac{\tau}{G}$

$\frac{\tau}{R} = \frac{G\theta}{L}\left(= \frac{\tau'}{r}\right)$ where τ' is the shear stress at any radius r.

Stresses: Let us consider a small strip of radius r and thickness dr which is subjected to shear stress τ'.



The force set up on each element = stress x area i.e $\tau' = \dfrac{G\theta . r}{L}$

we get $T = \int_0^R 2\pi \dfrac{G\theta}{L} . r^3 dr$

$T = \dfrac{2\pi G\theta}{L} \int_0^R r^3 dr$

$= \dfrac{2\pi G\theta}{L} . \left[\dfrac{R^4}{4}\right]_0^R$

$= \dfrac{G\theta}{L} . \dfrac{2\pi R^4}{4}$

$= \dfrac{G\theta}{L} . \dfrac{\pi R^4}{2}$

$= \dfrac{G\theta}{L} . \left[\dfrac{\pi d^4}{32}\right]$ now substituting $R = d/2$

$= \dfrac{G\theta}{L} . J$

since $\dfrac{\pi d^4}{32} = J$ the polar moment of inertia

or $\dfrac{T}{J} = \dfrac{G\theta}{L}$ ......(2)

if we combine the equation no.(1) and (2) we get $\boxed{\dfrac{T}{J} = \dfrac{\tau'}{r} = \dfrac{G.\theta}{L}}$

The total torque T on the section, will be the sum of all the contributions. Since τ' is a function of

radius so writing down τ' in terms of r from the equation (1). Because it varies with r

Where

T = applied external Torque, which is constant over Length L; J =

Polar moment of Inertia

$= \dfrac{\pi d^4}{32}$ for solid shaft

$= \dfrac{\pi (D^4 - d^4)}{32}$ for a hollow shaft.

[D =

Outside diameter ; d = inside

diameter] G = Modules of rigidity (or Modulus of elasticity in shear)

□ = It is the angle of twist in radians on a length L.

Tensional Stiffness: The tensional stiffness k is defined as the torque per radius twist i.e, k = T / □ = GJ / L

TORSION OF HOLLOW SHAFTS:

From the torsion of solid shafts of circular x – section , it is seen that only the material at the outer surface of the shaft can be stressed to the limit assigned as an

allowable working stresses. All of the material within the shaft will work at a lower stress and is not being used to full capacity. Thus, in these cases where the weight reduction is important, it is advantageous to use hollow shafts. In discussing the torsion of hollow shafts the same assumptions will be made as in the case of a solid shaft. The general torsion equation as we have applied in the case of torsion of solid shaft will hold good

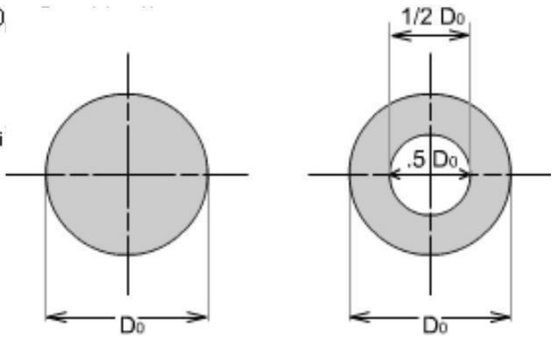
Hence by examining the equation (1) and (2) it may be seen that the □ m in the case of hollow sh aft is 6.6% larger than in the case of a solid shaft having the same outside diameter. **Reduction in weight:**

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G.\theta}{l}$$

For the hollow shaft

$$J = \frac{\pi(D_0^4 - d_i^4)}{32}$$ where D

Let d_i



$$\tau_{max^m}\Big|_{solid} = \frac{16T}{\pi D_0^3}$$

$$\tau_{max^m}\Big|_{hollow} = \frac{T.D_0/2}{\frac{\pi}{32}(D_0^4 - d_i^4)}$$

$$= \frac{16T.D_0}{\pi D_0^4\left[1-(d_i/D_0)^4\right]}$$

$$= \frac{16T}{\pi D_0^3\left[1-(1/2)^4\right]} = 1.066.\frac{16T}{\pi D_0^3} \qquad (2)$$

Considering a solid and hollow shafts of the same length 'l' and density '

Weight of hollow shaft

$$= \left[\frac{\pi D_0^2}{4} - \frac{\pi(D_0/2)^2}{4}\right] l \times \rho$$

$$= \left[\frac{\pi D_0^2}{4} - \frac{\pi D_0^2}{16}\right] l \times \rho$$

$$= \frac{\pi D_0^2}{4}\left[1 - 1/4\right] l \times \rho$$

$$= 0.75\frac{\pi D_0^2}{4} l \times \rho$$

$$\text{Weight of solid shaft} = \frac{\pi D_0^2}{4}.l.\rho$$

$$\text{Reduction in weight} = (1-0.75)\frac{\pi D_0^2}{4} l \times \rho$$

$$= 0.25\frac{\pi D_0^2}{4} l \times \rho$$

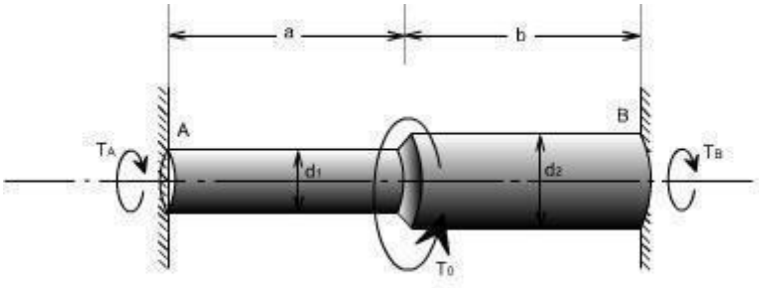
' with $d_i = 1/2\ D_o$ **Illustrative Examples :**

Problem 1

Hence the reduction in weight would be just 25%.

A stepped solid circular shaft is built in at its ends and subjected to an externally applied torque. T_0 at the shoulder as shown in the figure. Determine the angle of along the entire of the shoulder section where T_0 is length of the beam.

rotation □ ›plied ?



Closed Coiled helical Spring

Closed Coiled helical springs subjected to axial loads:

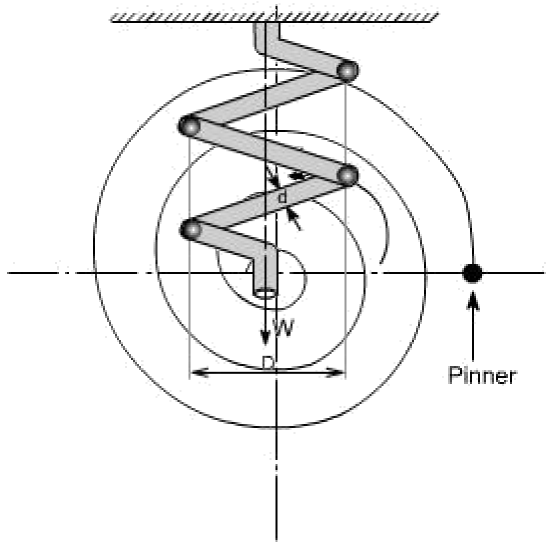
Definition: A spring may be defined as an elastic member whose primary function is to deflect or distort under the action of applied load; it recovers its original shape when load is released. or

Springs are energy absorbing units whose function is to store energy and to restore it slowly or rapidly depending on the particular application. **Important types of springs are:**

There are various types of springs such as

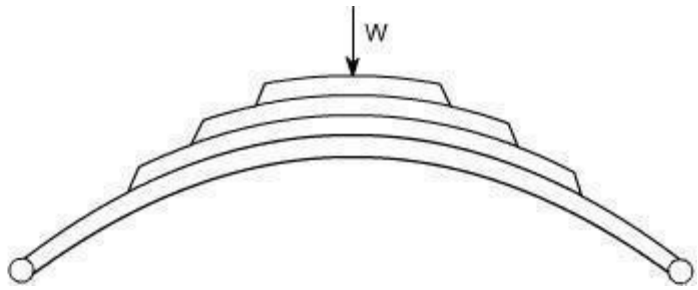
(i) helical spring: They are made of wire coiled into a helical form, the load being applied along the axis of the helix. In these type of springs the major stresses is torsional shear stress due to twisting. They are both used in tension and compression.

(ii) Spiral springs: They are made of flat strip of metal wound in the form of spiral and loaded in torsion.



(iv) Leaf springs: They are composed of flat bars of varying lengths clamped together so as to obtain greater efficiency . Leaf springs may be full elliptic, semi elliptic or cantilever types, In these type of springs the major stresses which come into picture are tensile & compressive.

These type of springs are used in the automobile suspension system.



Uses of springs :

(a) To apply forces and to control motions as in brakes and clutches.

(b) To measure forces as in spring balance.

(c) To store energy as in clock springs.

(d) To reduce the effect of shock or impact loading as in carriage springs.

(e) To change the vibrating characteristics of a member as inflexible mounting of motors. **Derivation of the Formula :**

In order to derive a necessary formula which governs the behaviour of springs, consider a closed coiled spring subjected to an axial load W.

Let

W = axial load

D = mean coil diameter

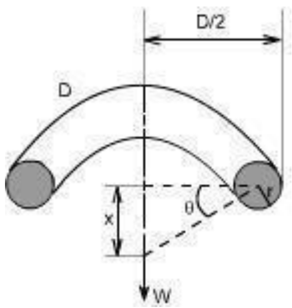
d = diameter of spring wire n =

number of active coils

C = spring index = D / d For circular wires l =
length of sprin g wire G = modulus
of rigidity x
= deflection of spring q =

Angle of twist when the spring is being subjected to an axial load to the wire of the spring gets be twisted like a shaft.
If q is the total angle of twist along the wire and x is the deflection of spring under the action of load W along the axis of the coil, so that x = D / 2 . <

again l = < D n [consider ,one half turn of a close coiled helical spring]



Assumptions: (1) The Bending & shear effects may be neglected

(2) For the purpose of derivation of formula, the helix angle is considered to be so

small that it may be neglected.

Any one coil of a such a spring will be assumed to lie in a plane which is nearly < r to the axis of the spring. This requires that adjoining coils be close together. With this limitation, a section taken perpendicular to the axis the spring rod becomes nearly vertical. Hence to maintain equilibrium of a segment of the spring, only a shearing force V = F and Torque T = F. r are required at any X – section. In the analysis of springs it is customary to assume that the shearing stresses caused by the direct shear force is uniformly distributed and is negligible so applying the torsion formula.

Using the torsion formula i.e

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G.\theta}{l}$$

and substitituting $J = \frac{\pi d^4}{32}; T = w.\frac{d}{2}$

$$\theta = \frac{2.x}{D}; l = \pi D.x$$

$$K = \frac{4c-1}{4c-4} + \frac{0.615}{c}$$

<u>SPRING DE FLECTION</u>

$$\frac{w.d/2}{\frac{\pi d^4}{32}} = \frac{G.2x/D}{\pi D.n}$$

Thus,

$$x = \frac{8w.D^3.n}{G.d^4}$$

Spring striffness: The stiffness is defined as the load per unit deflection therefore

$$k = \frac{w}{x} = \frac{w}{\dfrac{8w.D^3.n}{G.d^4}}$$

Therefore

$$k = \frac{G.d^4}{8.D^3.n}$$

Shear stress

$$\frac{w.d/2}{\dfrac{\pi d^4}{32}} = \frac{\tau_{max^m}}{d/2}$$

$$\text{or } \tau_{max^m} = \frac{8wD}{\pi d^3}$$

WAHL'S FACTOR :

In order to take into account the effect of direct shear and change in coil curvature a stress factor is defined, which is known as Wahl's factor

K = Wahl' s factor and is defined as Where C
= spring index
= D/d

if we take into account the Wahl's factor than the formula for the shear stress

becomes $\tau_{max^m} = \dfrac{16.T.k}{\pi d^3}$

Strain Energy : The strain energy is defined as the energy which is stored within a material when the work has been done on the material.

$$x = \frac{8wD^3.n}{G.d^4}$$

on substituting the relevant parameters we get

$$50 = \frac{8.5000.(0.0314\,d^3)^3.8}{83{,}000.d^4}$$

$$d = 13.32\,mm$$

In the case of a spring the strain energy would be due to bending and the strain energy due to bending is given by the expansion

$$U = \frac{T^2 L}{2EI}$$

$$L = \pi D n$$

$$I = \frac{\pi d^4}{64}$$

so after substitution we get

$$U = \frac{32 T^2 D n}{E d^4}$$

Example: A close coiled helical spring is to carry a load of 5000N with a deflection of 50 mm and a maximum shearing stress of 400 N/mm^2 .if the number of active turns or active coils is 8.Estimate the following:

(i) wire diameter

(ii) mean coil diameter

(iii) weight of the spring.

Assume G = 83,000 N/mm^2 ; < = 7700 kg/m^3

solution :

(i) for wire diametre if W is the axial load, then

$$\frac{\frac{w.d/2}{\frac{\pi d^4}{32}}}{} = \frac{\tau_{max^m}}{d/2}$$

$$D = \frac{400}{d/2} \cdot \frac{\pi d^4}{32} \cdot \frac{2}{W}$$

$$D = \frac{400 . \pi d^3 . 2}{5000.16}$$

$$D = 0.0314\, d^3$$

Futher, deflection is given as

Therefore,

D = .0314 x (13.317)^{3}mm

=74.15mm D

= 74.15 mm

Weight

mass or weight = volume. density

= area.length of the spring.density of spring material

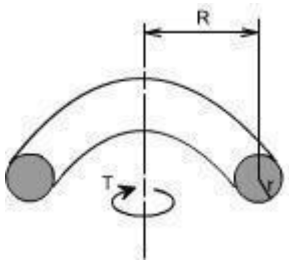
$= \frac{\pi d^2}{4} . \pi D n . \rho$

On substituting the relevant parameters we get

Weight = 1.996 kg

≈ 2.0 kg

Close – coiled helical spring subjected to axial torque T or axial couple.



In this case the material of the spring is subjected to pure bending which tends to reduce Radius R of the coils. In this case the bending moment is constant through out the spring and is equal to the applied axial Torque T. The stresses i.e. maximum

$$\sigma_{max} = \frac{M.y}{I} = \frac{T.d/2}{\frac{\pi d^4}{64}}$$

$$\sigma_{max} = \frac{32T}{\pi d^3}$$

bending stress may thus be determined from the bending theory.

Deflection or wind – up angle:

Under the action of an axial torque the deflection of the spring becomes the "wind – up" angle of the spring which is the angle through which one end turns relative to the

other. This will be equal to the total change of slope along the wire, according to area –

moment theorem

$$\theta = \int_0^L \frac{M\,dL}{EI} \quad \text{but } M = T$$

$$= \int_0^L \frac{T.dL}{EI} = \frac{T}{EI}\int_0^L dL$$

Thus, as 'T' remains constant

$$\theta = \frac{T.L}{EI}$$

Futher

$$L = \pi D.n$$

$$I = \frac{\pi d^4}{64}$$

Therefore, on substitution,the value of θ obtained is

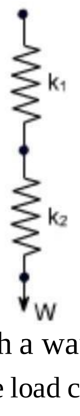
$$\boxed{\theta = \frac{64T.D.n}{E.d^4}}$$

Springs in Series: If two springs of different stiffness are joined endon and carry a common load W, they are said to be connected in series and the combined stiffness and deflection are given by the following equation.

$$\frac{W}{k} = x_1 + x_2 = \frac{W}{k_1} + \frac{W}{k_2}$$

or

$$\boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}}$$



Springs in parallel: If the two spring are joined in such a way that they have a common deflection „x' ; then they are said to be connected in parallel.In this care the load carried is shared between the two springs and total load $W = W_1 + W_2$

$$x = \frac{W}{k} = \frac{W_1}{k_1} = \frac{W_2}{k_2}$$

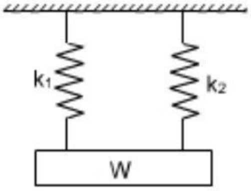
Thus $W_1 = \frac{Wk_1}{k}$

$$W_2 = \frac{Wk_2}{k}$$

Futher

$$W = W_1 + W_2$$

thus $\boxed{k = k_1 + k_2}$



UNIT -2

Columns and Struts

Introduction:

Structural members which carry compressive loads may be divided into two broad categories depending on their relative lengths and cross -sectional dimensions. **Columns:**

Short, thick members are generally termed columns and these usually fail by crushing when the yield stress of the material in compression is exceeded. **Struts:**

Long, slender columns are generally termed as struts, they fail by buckling some time before the yield stress in compression is reached. The buckling occurs owing to one the following reasons. (a). the strut may not be perfectly straight initially.

(b). the load may not be applied exactly along the axis of the Strut.

(c). one part of the material may yield in compression more readily than others owing to some lack of uniformity in the material properties through out the strut.

In all the problems considered so far we have assumed that the deformation to be both progressive with increasing load and simple in form i.e. we assumed that a member in simple tension or compression becomes progressively longer or shorter but remains straight. Under some circumstances however, our assumptions of progressive and simple deformation may no longer hold good and the member become unstable. The term strut and column are widely used, often interchangeably in the context of buckling of slender members.]

At values of load below the buckling load a strut will be in stable equilibrium where the displacement caused by any lateral disturbance will be totally recovered when the disturbance is removed. At the buckling load the strut is said to be in a state of neutral equilibrium, and theoretically it should than be possible to gently deflect the strut into a simple sine wave provided that the amplitude of wave is kept small.

Theoretically, it is possible for struts to achieve a condition of unstable equilibrium with loads exceeding the buckling load, any slight lateral disturbance then causing failure by buckling, this condition is never achieved in practice under static load conditions. Buckling occurs immediately at the point where the buckling load is reached, owing to the reasons stated earlier.

The resistance of any member to bending is determined by its flexural rigidity EI and is The quantity I may be written as $I = Ak^2$, Wh ere I = area of moment of inertia A = area of the cross -section k = radius of gyration.

The load per unit area which the member can withstand is therefore related to k. There will be two principal moments of inertia, if the least of these is taken then the ratio

Is called the slenderness ratio. It's numerical value indicates whether the member falls into the class

$\frac{l}{k}$ i.e. $\frac{\text{length of member}}{\text{least radius of gyration}}$

of columns or struts.

$EI\, M_C = -Py$

Futher,we know that

$$EI\frac{d^2y}{dx^2} = M$$

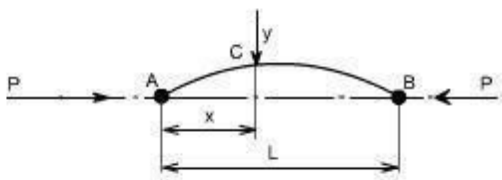
$$EI\frac{d^2y}{dx^2} = -P.y = M$$

Euler's Theory : The struts which fail by buckling can be analyzed by Euler's theory. In the following sections, different cases of the struts have been analyzed. **Case A: Strut with pinned ends:**

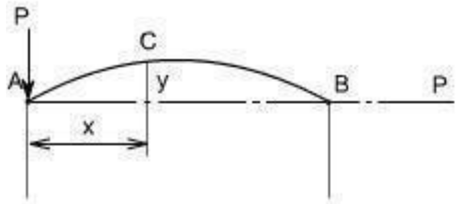
Consider an axially loaded strut, shown below, and is subjected to an axial load „P' this load „P' produces a deflection „y' at a distance „x' from one end.

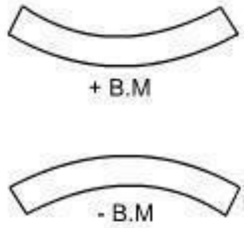
Assume that the ends are either pin jointed or rounded so that there is no moment at either end.

Assumption:



The strut is assumed to be initially straight, the end load being applied axially through centroid.





In this equation „M' is not a function „x'. Therefore this equation can not be integrated directly as has been done in the case of deflection of beams by integration method.

Thus,

$$EI\frac{d^2y}{dx^2} + Py = 0$$

Though this equation is in „y' but we can't say at this stage where the deflection would be maximum or min imum.

So the above differential equation can be arranged in the following

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = 0$$ form

Let us define a operator D = d/dx
$(D^2 + n^2)\, y = 0$ where $n^2 = P/EI$

This is a second order differential equation which has a solution of the form consisting of complimentary function and particular integral but for the time being we are interested in the complementary solution only[in this P.I = 0; since the R.H.S of Diff. equation = 0] Thus y = A cos (nx) + B sin (nx) Where A and B are some constants.

Therefore

$$y = A\cos\sqrt{\frac{P}{EI}}\,x + B\sin\sqrt{\frac{P}{EI}}\,x$$

In order to evaluate the constants A and B let us apply the boundary conditions,

(i) at x = 0; y = 0

(ii) at x = L ; y = 0

Applying the first boundary condition yields A = 0. Applying the second boundary condition gives

$$B\sin\left(L\sqrt{\frac{P}{EI}}\right) = 0$$

Thus either $B = 0$, or $\sin\left(L\sqrt{\frac{P}{EI}}\right) = 0$

if B=0,that y0 for all values of x hence the strut has not buckled yet.Therefore,the solution required is

$$\sin\left(L\sqrt{\frac{P}{EI}}\right) = 0 \quad \text{or} \quad \left(L\sqrt{\frac{P}{EI}}\right) = \pi \text{ or } nL = \pi$$

$$\text{or } \sqrt{\frac{P}{EI}} = \frac{\pi}{L} \quad \text{or} \quad P = \frac{\pi^2 EI}{L^2}$$

From the above relationship the least value of P which will cause the strut to buckle, and it is called the " **Euler Crippling Load** " $P_e$from which w obtain.

$$\boxed{P_e = \frac{\pi^2 EI}{L^2}}$$

It may be noted that the value of I used in this expression is the least moment of inertia

It should be noted that the other solutions exists for the equation

$$\sin\left(l\sqrt{\frac{P}{EI}}\right) = 0 \qquad \text{i.e. } \sin nL = 0$$

The interpretation of the above analysis is that for all the values of the load P, other than those which make sin nL = 0; the strut will remain perfectly straight since y = B sin nL = 0

For the particular value of

$$P_e = \frac{\pi^2 EI}{L^2}$$

$\sin nL = 0 \quad \text{or } nL = \pi$

Therefore $n = \frac{\pi}{L}$

Hence $y = B \sin nx = B \sin \frac{\pi x}{L}$

Then we say that the strut is in a state of neutral equilibrium, and theoretically any deflection which it suffers will be maintained. This is subjected to the limitation that „L' remains sensibly constant and in practice slight increase in load at the critical value will cause the deflection to increase appreciably until the material fails by yielding.

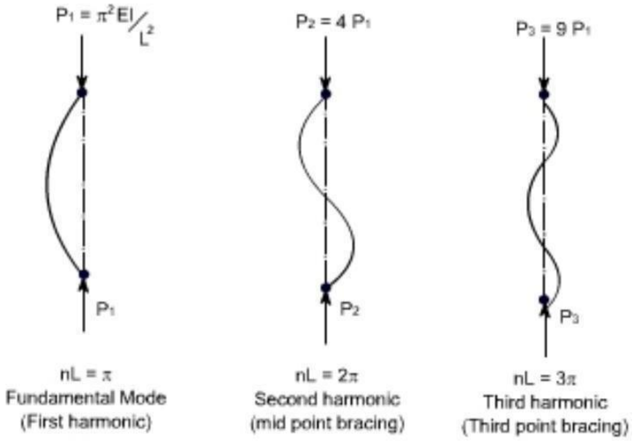
Further it should be noted that the deflection is not proportional to load, and this applies to all strut problems; like wise it will be found that the maximum stress is not proportional to load.

The solution chosen of nL = < is just one particular solution; the solutions nL= 2< , 3< , 5< etc are equally valid mathematically and they do, infact, produce values of

„P_e' which are equally valid for modes of buckling of strut different from that of a simple bow. Theoretically therefore, there are an infinite number of values of P_e , each corresponding with a different mode of buckling.

The value selected above is so called the fundamental mode value and is the lowest critical load producing the single bow buckling condition.

The solution nL = 2< produces buckling in two half – waves, 3< in three half-waves etc.



$$L\sqrt{\frac{P}{EI}} = \pi \quad \text{or} \quad P_1 = \frac{\pi^2 EI}{L^2}$$

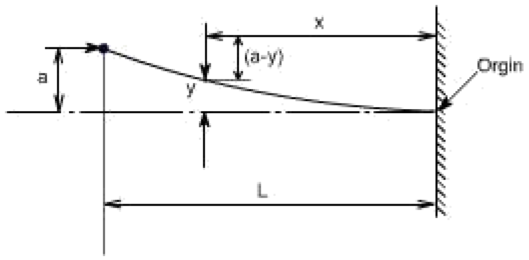
$$\text{If } L\sqrt{\frac{P}{EI}} = 2\pi \text{ or } P_2 = \frac{4\pi^2 EI}{L^2} = 4P_1$$

$$\text{If } L\sqrt{\frac{P}{EI}} = 3\pi \text{ or } P_3 = \frac{9\pi^2 EI}{L^2} = 9P_1$$

If load is applied sufficiently quickly to the strut, then it is possible to pass through the fundamental mode and to achieve at least one of the other modes which are theoretically possible. In practical loading situations, however, this is rarely achieved since the high stress associated with the first critical condition generally ensures immediate collapse.

struts and columns with other end conditions: Let us consider the struts and columns h aving different end conditions

Case b: One end fixed and the other free:



writing down the value of bending moment at the point C

$B.\ M|_c = P(a - y)$

Hence, the differential equation becomes,

$$EI\frac{d^2y}{dx^2} = P(a - y)$$

On rearranging we get

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{Pa}{EI}$$

Let $\frac{P}{EI} = n^2$

Hence in operator form, the differential equation reduces to $(D^2 + n^2)\ y = n^2a$

The solution of the above equation would consist of complementary solution and particular solution, therefore $y_{gen} = A\cos(nx) + \sin(nx) + P.\ I$ where

P.I = the P.I is a particular value of y which satisfies the differential equation Hence $y_{P.I} = a$ Therefore the complete solution becomes $Y = A\cos(nx) + B\sin(nx) + a$

Now imposing the boundary conditions to evaluate the constants A and B

(i) at

x = 0; y = 0

This yields A = -a

(ii) at x = 0; dy/dx = 0 This yields B = 0

Hence y = < a cos(nx)

+ a Futher, at x = L; y = a

Therefore a = - a cos(nx) + a or 0 = cos(nL)

Hence the least solution would be

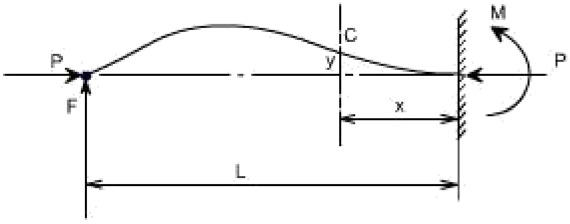
$nL = 2\pi$

$nL = \sqrt{\frac{P}{EI}}\, L = 2\pi$. Thus the buckling load or crippling load is

Now the fundamental mode of buckling in this case would be

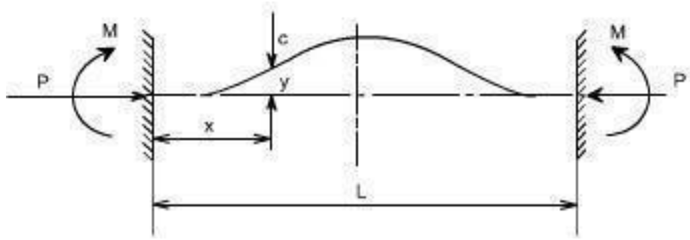
$\sqrt{\frac{P}{EI}}\, L = \frac{\pi}{2}$, Therefore, the Euler's crippling load is given as $P_e = \frac{4\pi^2 EI}{L^2}$

$$P_e = \frac{\pi^2 EI}{4L^2}$$



Case 3

Strut with fixed ends:



Due to the fixed end supports bending moment would also appears at the supports, since this is the property of the support.

Bending Moment at point C = M – P.y

One end fixed, the other pinned

In order to maintain the pin-joint on the horizontal axis of the unloaded strut, it is necessary in this case to introduce a vertical load F at the pin. The moment of F about the built in end then balances the fixing moment.

With the origin at the built in end, the B,M at C is given as

$$EI\frac{d^2y}{dx^2} = -Py + F(L-x)$$

$$EI\frac{d^2y}{dx^2} + Py = F(L-x)$$

Hence

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{F}{EI}(L-x)$$

In the operator form the equation reduces to

$$\left(D^2 + n^2\right)y = \frac{F}{EI}(L-x)$$

$$y_{particular} = \frac{F}{n^2EI}(L-x) \text{ or } y = \frac{F}{P}(L-x)$$

The full solution is therefore

$$y = A\,Cos\,mx + B\,Sin\,nx + \frac{F}{P}(L-x)$$

The boundry conditions relevants to the problem are at x=0;y=0

$$\text{Hence } A = -\frac{FL}{P}$$

$$\text{Also at } x = 0; \frac{dy}{dx} = 0$$

$$\text{Hence } B = \frac{F}{nP}$$

$$\text{or } y = -\frac{FL}{P}Cos\,nx + \frac{F}{nP}Sin\,nx + \frac{F}{P}(L-x)$$

$$y = \frac{F}{nP}\left[Sin\,nx - nL\,Cos\,nx + n(L-x)\right]$$

Also when x = L ; y = 0

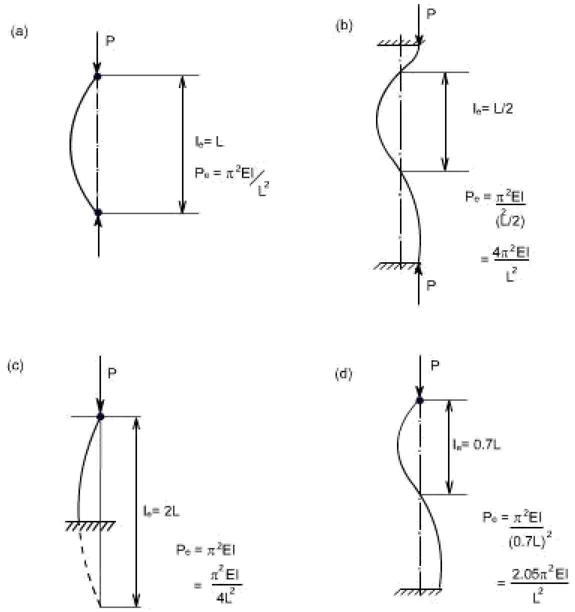
Therefore nL Cos nL = Sin nL

The lowest value of nL (neglecting zero) which satisfies this condition and which therefore produces the fundamental buckling condition is nL = 4.49radian

$$\text{or} \sqrt{\frac{P}{EI}}\, L = 4.49$$

$$\frac{P_e}{EI} L^2 = 20.2$$

$$P_e = \frac{2.05\pi^2\, EI}{L^2}$$



Equivalent Strut Length:

$$\text{Euler's stress, } \sigma_e = \frac{P_e}{A} = \frac{\pi^2 EI}{Al^2}$$

But, $I = Ak^2$

Having derived the results for the buckling load of a strut with pinned ends the Euler loads for other end conditions may all be written in the same form.

$$\sigma_e = \frac{\pi^2 E}{\left(\frac{l}{k}\right)^2}$$

$$\text{i.e. } P_e = \frac{\pi^2 EI}{L^2}$$

Where L is the equivalent length of the strut and can be related to the actual length of the strut depending on the end conditions.

The equivalent length is found to be the length of a simple bow(half sine wave) in each of the strut deflection curves shown. The buckling load for each end condition shown is then readily obtained. The use of equivalent length is not restricted to the Euler's theory and it will be used in other derivations later.

The critical load for columns with other end conditions can be expressed in terms of the critical load for a hinge d column, which is taken as a fundamental case.

For case(c) see the figure, the column or strut has inflection points at quarter points of its unsupported length. Since the bending moment is zero at a point of inflection, the freebody diagram would indicates that the middle half of the fixed ended is equivalent to a hinged column having an effective length $L_e = L / 2$. The four different cases which we have considered so far are:

(a) Both ends pinned (c) One end fixed, other free

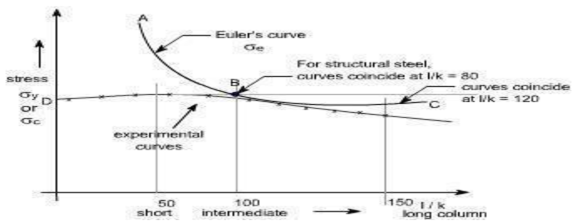
(b) Both ends fixed (d) One end fixed and other pinned

Limitations of Euler's Theory :

In practice the ideal conditions are never [i.e. the strut is initially straight and the end load being applied axially through centroid] reached. There is always some eccentricity and initial curvature present. These factors needs to be accommodated in the required formula's.

It is realized that, due to the above mentioned imperfections the strut will suffer a deflection which increases with load and consequently a bending moment is introduced which causes failure before the Euler's load is reached. Infact failure is by stress rather than by buckling and the deviation from the Euler value is more marked as the slenderness-ratio l/k is reduced. For values of l/k < 120 approx, the error in applying the Euler theory is too great to allow of its use. The stress to cause buckling from the Euler formula for the pin ended strut is

A plot of < e versus l / k ratio is shown by the curve ABC.



Allowing for the imperfections of loading and strut, actual values at failure must lie within and below line CBD.

Other formulae have therefore been derived to attempt to obtain closer agreement between the actual failing load and the predicted value in this particular range of slenderness ratio i.e.l/k=40 to l/k=100.

***(a)** Straight – line formulae :*

The permissible load is given by the formulae

$P = \sigma_y A\left[1 - n\left(\frac{l}{k}\right)\right]$ Where the value of index „n' depends on the material used and the end conditions.

(b) Johnson parabolic formulae : The Johnson parabolic formulae is defined as

$$P = \sigma_y A\left[1 - b\left(\frac{l}{k}\right)^2\right]$$ where the value of index „b' depends on the end conditions.

(a) *Rankine Gordon Formulae :*

$$\frac{1}{P_R} = \frac{1}{P_e} + \frac{1}{P_c}$$

Where P_e = Euler crippling load

P_c = Crushing load or Yield point load in Compression P_R =
Actual load to cause failure or Rankine load
Since the Rankine formulae is a combination of the Euler and crushing load for a strut.

For a very short strut P_e is very large hence $1/P_e$ would be large so that $1/P_e$ can be neglected.

$$\frac{1}{P_R} = \frac{1}{P_e} + \frac{1}{P_c}$$

Thus $P_R = P_c$, for very large struts, P_e is very small so $1/P_e$ would be large and $1/P_c$ can be neglected ,hence $P_R = P_e$

The Rankine formulae is therefore valid for extreme values of 1/k.It is also found to be fairly accurate for the intermediate values in the range under consideration. Thus rewriting the formula in te rms of stresses, we have

$$a = \frac{\sigma_y}{\pi^2 EI}$$

$$\frac{1}{\sigma A} = \frac{1}{\sigma_e A} + \frac{1}{\sigma_y A}$$

$$\frac{1}{\sigma} = \frac{1}{\sigma_e} + \frac{1}{\sigma_y}$$

$$\frac{1}{\sigma} = \frac{\sigma_e + \sigma_y}{\sigma_e . \sigma_y}$$

$$\sigma = \frac{\sigma_e . \sigma_y}{\sigma_e + \sigma_y} = \frac{\sigma_y}{1 + \frac{\sigma_y}{\sigma_e}}$$

For struts with both ends pinned

$$\sigma_e = \frac{\pi^2 E}{\left(\frac{l}{k}\right)^2}$$

$$\sigma = \frac{\sigma_y}{1 + \frac{\sigma_y}{\pi^2 E}\left(\frac{l}{k}\right)^2}$$

.

$$\sigma = \frac{\sigma_y}{1 + a\left(\frac{l}{k}\right)^2}$$

Where and the value of „a' is found by conducting experiments on various materials. Theoretically, but having a value normally found by experiment for various materials. This will take into account other types of end conditions.

UNIT-3
DIRECT AND BENDING STRESSES

AXIAL LOAD AND BENDING MOMENT

Figure .shows an isometric view of a rectangular section loaded with an axial load ‘*P*’, applied long its vertical axis through the centroid of the horizontal section. We say such a member is subjected to uniform compressive stress or direct compressive stress of magnitude *P*/*A*, where ‘*A*’ is the area of the horizontal cross-section.

After studying this unit, you should be able to

- calculate the stresses for different eccentrically loaded cross-section,
- explain the Middle Third Rule for no tension condition,
- analyse the effect of wind forces and their stress distribution pattern onmasonry walls, pillars and tall chimneys, and
- design sections for members carrying direct compressive force and bendingstresses.

forces and Stressesin Beams If the point of application of load *P* is displaced by a small distance '*e*' from the axis, asshown in Figure 7.1(b), the distribution of stress in the member is considerably affected.(Here, '*e*' is called the eccentrically of loading.)

Figure 7.1(c) shows the elevation of the member is viewed from the face *ABCD*. Due to the eccentric load *P*, the member is distorted or bend, as shown in Figure 7.1(d). The left half portion and so the side *AD* will be in tension and the right half and so the side *BC* will be in compression, thereby making the central axis of the member as neutral axric load, the member is subjected to direct stressaccompanied by the bending stress.

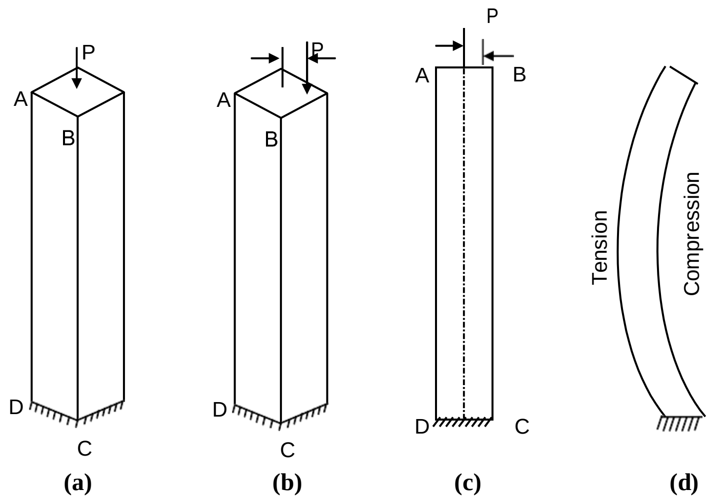
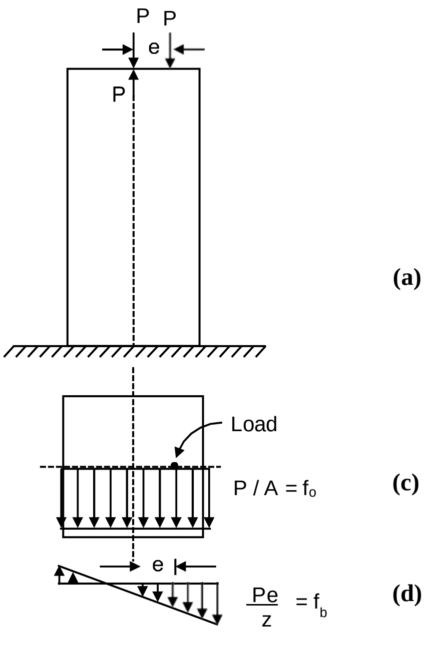


Figure .1

ECCENTRICALLY LOADED SECTIONS

Load Acting Eccentric to One Axis

In order to study the effect of eccentric load more closely, let us consider a short axial member, loaded with load *P*, placed at a distance '*e*' from the centroidal vertical axis through the centroid of the section, as shown in Figure



Figure

Along the vertical axis, introduce two equal and opposite forces, each equal to load P. Their introduction obviously makes to difference to the loading of the member, as they cancel out each other. However, if the upward force P is considered along with at a distance e from each other, from a clockwise couple of magnitude $P \times e$, the effect of which is to produce bending stress in the member. The remaining central downward force$P$, produces a direct compressive stress f_0, of magnitude P/A as usual. Hence, we can conclude that an eccentric load produces direct compressive stress as well as the bending stress.

The bending couple $P \times e$ will cause longitudinal tensile and compressive stresses. The fibre stress due to bending f_0, at any distance 'y' from the neutral axis is given by,

$$f_b = \frac{M}{I_{xx}} \times y = \frac{P \times e \times y}{I_{xx}} \quad \text{(tensile or compressive)} \qquad \text{. . . (.1}$$

Hence, the total stress at any section is given by

$$f = f_0 \pm f_b = \frac{P}{A} \pm \frac{P \times e \times y}{I_{xx}} \qquad \text{. . . (.2)}$$

$$f = \frac{P}{A} \pm \frac{M}{Z_{xx}}$$

[where $P \times e = M$ and $\frac{I_{xx}}{y} = Z_{xx}$ (the section modulus)].

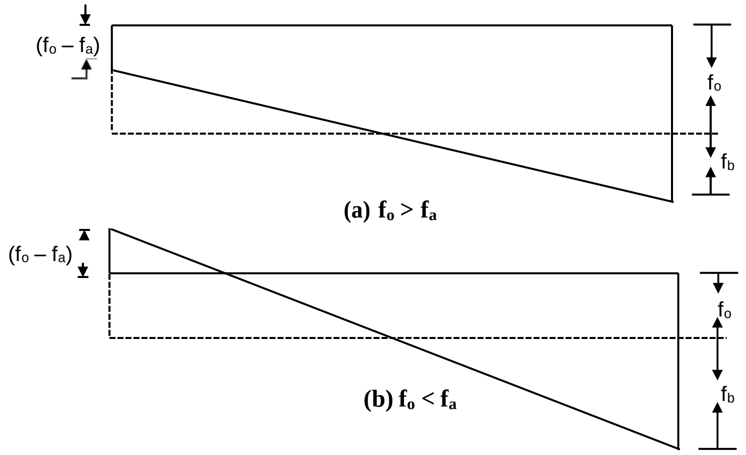
The extreme fibre stresses are given by,

$$f_{max} = f_0 + f_b = \frac{P}{A} + \frac{M}{Z_{xx}}$$

and

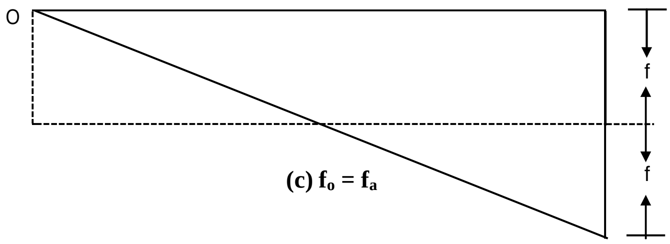
$$f_{min} = f_0 - f_b = \frac{P}{A} - \frac{M}{Z_{xx}} \qquad \text{. . . (.3)}$$

If f_0 is greater than f_b, the stress throughout the section will be of the same sign. If however, f_0 is less than f_b, the stress will change sign, being partly tensile and partly compressive across the section. Thus, there can be three possible stress distributions as shown in Figures 7.3(a), (b) and (c). You can observe from Figure 7.3(c), that when $f_0 = f_b$, $f_{max} = 2f_0$ and $f_{min} = 0$.



(a) $f_o > f_a$

(b) $f_o < f_a$



(c) $f_o = f_a$

Figure .3

Load Acting Eccentric to Both Axis

If the axial load *P* is placed eccentric to both *x*-axis and *y*-axis as shown in Figure .4, then the system can be assumed to consist of

(a) a direct compressive force *P* acting at the centroid,

(b) a couple $P \times e_x$ about the *x*-axis, and

(c) a couple $P \times e_y$ about the *y*-axis.

As seen for the case of load acting eccentric to one axis, the stress at any point can be written as

$$f = f_0 + f_{b1} \pm f_{b2}$$

$$= \frac{P}{A} \pm \frac{P \times e_x}{I_{xx}} \times y \pm \frac{P \times e_y}{I_{xx}} \times x$$

$$= \frac{P}{A} \pm \frac{M_{xx}}{Z_{xx}} \pm \frac{M_{yy}}{Z_{yy}} \qquad \ldots (.4)$$

The maximum or minimum fibre stress will occur at the corner point *A*, *B*, *C* or *D* in Figure .4 for the symmetrical section.

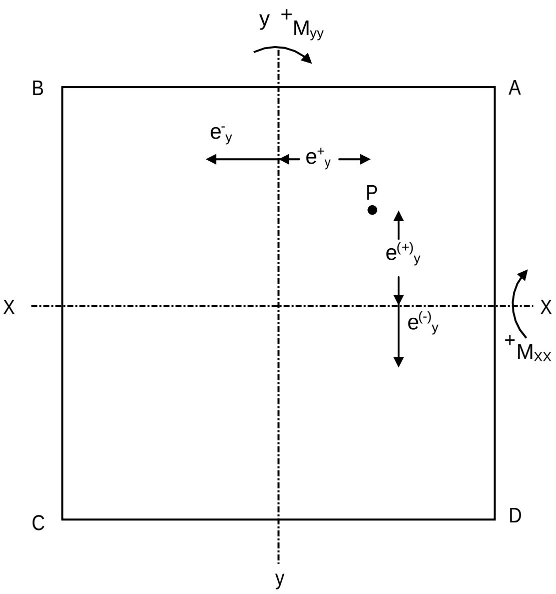


Figure .4

Neutral Axis

If you refer to Figure .1(d), the side *AD* will be in tension while the side *BC* will be in compression. The fibres along the side *AD* elongates, while the fibres along the side *BC* shortens. Also there exists one fibre in between these two faces which neither have extension nor compression. This layer is called the neutral layer. The line of intersection of neutral layer with the plane of cross-section, of the member is called the neutral axis. For members subjected to bending only, the neutral axis passes through the centroid of the section. At the neutral axis, the stress will be zero.

CONDITION FOR NO TENSION IN THESECTION

Middle Third Rule

In Figure (b) $f_0 < f_b$ and therefore, stress changes sign, being partly tensile and partly compressive across the section. In masonry and concrete structures, the development of tensile stress in the section is not desirable, as they are weak in tension. This limits the eccentricity *e* to a certain value which will be investigated now for different sections.

In order that the stress may not change sign from compressive to tensile, we have

$$f_0 \geq f_b$$

i.e.

$$\frac{P}{A} \geq \frac{Pe}{I} \times \frac{d}{2}$$

$$\frac{P}{A} \geq \frac{Ped}{2AK^2}$$

or

$$e \leq \frac{2k^2}{d}$$

where, k = radius of gyration of the section with regard to N.A. and d is the depth of the section.

Thus, for no tension in the section, the eccentrically must not exceed $\frac{2k^2}{d}$.

Let us now take a rectangular section and find out the limiting value of e.

For a rectangular section of width b and depth d,

$$I = \frac{1}{12} bd^3 \text{ and } A = bd$$

$$k^2 = \frac{I}{A} = \frac{d^2}{12}$$

Substituting in Eq. (7.5), we get

$$e \leq \frac{2d^2}{d \times 12} \leq \frac{d}{6}$$

$\therefore$

$$I_{max} = \frac{d}{6}$$

The value of eccentricity can be on either side of the geometrical axis. Thus, the stress will be of the same sign throughout the section if the load line is within the middle third of the section.

In the case of rectangular section, the maximum intensities of extreme stresses are given by

$$f = \frac{P}{A} \pm \frac{Pl}{Z_{xx}} = \frac{P}{bd} \pm \frac{6pe}{bd^2}$$

$$= \frac{P}{bd} \left[1 \pm \frac{6e}{d} \right]$$

Core or Kernel of a Section

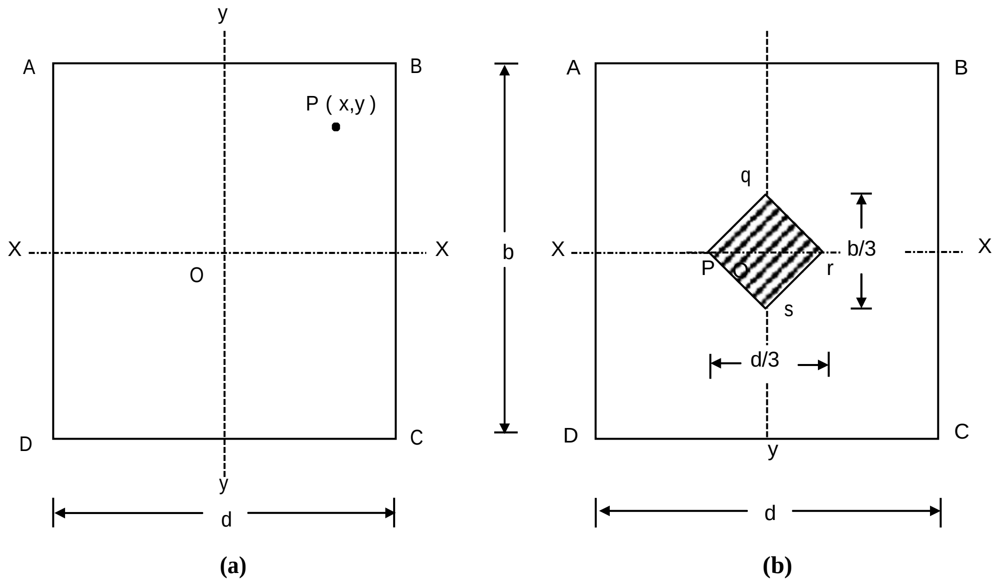
If the line of action of the stress is on neither of the centre lines of the section, the bending is unsymmetrical. However, there is certain area within the line of action of the force P must cut the cross-section if the stress is not to become tensile. This area we call it as 'core' or 'kernel' of the section. Let us calculate this for a rectangular section.

Rectangular Section

Let the point of application of the load P have the coordinates (x, y), with reference

to the axes show

n in Figure (a) in which x is positive when measured to right of O and y is positive when measured upwards.



Figure

The stress at any point have coordinates (x', y') will be

$$f = \frac{P}{bd} + \frac{P \times yy'}{\frac{1}{12} db^3} + \frac{P \times xx'}{\frac{1}{12} bd^3}$$

$$= \frac{12P}{bd}\left(\frac{1}{12} + \frac{yy'}{b^2} + \frac{xx'}{d^2}\right)$$

At D, $x' = -\frac{d}{2}$ and $y' = -\frac{b}{2}$ and, therefore, f will be minimum. Thus, at D, we have,

$$f = \frac{6P}{bd}\left(\frac{1}{6} - \frac{y}{b} - \frac{x}{d}\right)$$

The value of f reaches zero when

$$\frac{y}{b} + \frac{x}{d} = \frac{1}{6} \quad \text{or} \quad \frac{6y}{b} + \frac{6x}{d} = 1$$

Thus, the deviation of the load line is governed by the straight line of Eq., whose intercepts on the axes are respectively $\frac{b}{6}$ and $\frac{d}{6}$. This is true for the load line is the first quadrant. Similar limits will apply in other quadrants and the stress will be wholly compressive throughout the section, if the line of action of P will within the rhombus *pqrs* (Figure 7.5(b)), the diagonals of which are of length $\frac{d}{3}$ and $\frac{b}{3}$, respectively. This rhombus is called the core of the rectangular section.

STRESS DISTRIBUTION FOR DIFFERENTECCENTRICALLY LOADED SECTIONS

Circular Section

We have seen that for no tension,

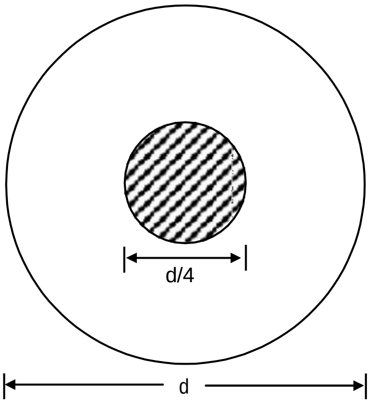
$$e \leq \frac{2k^2}{d}$$

For a solid circular section,

$$I = \frac{\pi d^4}{64}; \qquad A = \frac{\pi}{4} d^2$$

$$\therefore \qquad k^2 = \frac{I}{A} = \frac{d^2}{16}$$

$$\therefore \qquad e \le \frac{2}{d} \times \frac{d^2}{16} \le \frac{d}{8} \qquad \ldots (7.10)$$



Figure

Thus, in order that tension is not to be developed, the load line must fall within middle fourth of the section. The core in this case is a circle with the same centre and diameter $\frac{d}{4}$ as shown in Figure 7.6.

Hollow Section

For a hollow section, having external diameter D and internal diameter d,

$$I = \frac{\pi}{64} (D^4 - d^4); \text{ and } \quad A = \frac{\pi}{4} (D^2 - d^2)$$

$$\therefore \qquad k^2 = \frac{I}{A} = \frac{D^4 - d^4}{16 (D^2 - d^2)} = \frac{D^2 + d^2}{16}$$

$$\therefore \qquad e \le \frac{2}{D} \left(\frac{D^2 + d^2}{16} \right) = \frac{D^2 + d^2}{8D} \qquad \ldots (7.11)$$

The core for a hollow circular section is thus, a concentric circle of diameter

$$\left(\frac{D^2 + d^2}{4D} \right).$$

Structural Sections

Eq. (7.8) can be rewritten in the form,

$$f = \left(1 + \frac{yy'}{k_x^2} + \frac{xx'}{k_y^2}\right)$$

where k_x and k_y are radii of gyration of the area of section about the axes of x and y, respectively.

For zero stress at the point, we must have

$$\frac{yy'}{k_x^2} + \frac{xx'}{k_y^2} = -1$$

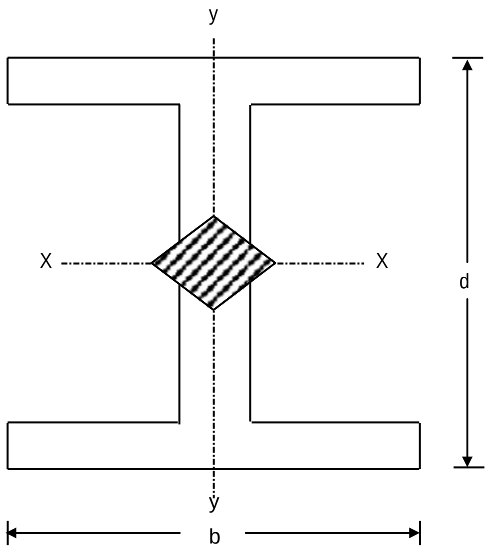


Figure 7.7

For an *I*-section, the four corners will be the limiting points, where $x' = \frac{b}{2}$ and $y' = \frac{d}{2}$.

Hence, we have

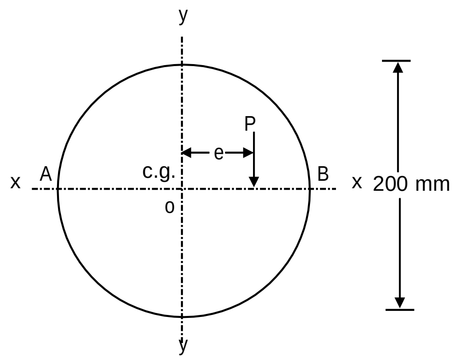
$$\frac{yd}{2k_x^2} + \frac{xb}{2k_y^2} = -1$$

$$\therefore \qquad y = \frac{k_x^2}{k_y^2} \times \frac{b}{d}\, x - \frac{2k_x^2}{d}$$

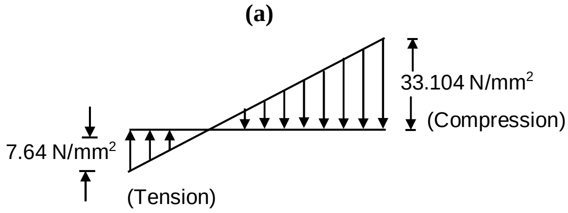
Eq. is the equation of the bounding line which limits the deviation of load from the centroid for no change in the sign of the stress, for *I*-section. The equations to the three other bounding lines will be similar, thus, forming a rhombus having the principal axis of *I*-section as diagonals, as shown in Figure .

Example .1

A cast iron column of 200 mm diameter carries a vertical load of 400 kN, at a distance of 50 mm from the centre. Determine the maximum and minimum stress developed in the section, along the diameter passing through the point of loading.



(a)



(b) Stress Distribution along the Diagonal

Solution

Vertical load, $P = 400 \text{ kN} = 400 \times 10^3$ N

Diameter of the section, $D = 200$ mm

Area of the section, $A = \frac{\pi}{4}(200)^2 = 31416 \text{ mm}^2$

Direct stress, $f_0 = \frac{P}{A} = \frac{4 \times 10^5}{31416} = 12.732 \text{ N/mm}^2$

Eccentricity, $e = 40$ mm

Bending moment, $M = P \times e = (400 \times 10^3)\ 40 = 16 \times 10^6$ N-mm

Section modulus, $z = \frac{\pi D^3}{32} = \frac{\pi\,(200)^3}{32} = 785.4 \times 10^3 \text{ mm}^3$

Bending stress, $f_b = \pm \frac{Pe}{Z} = \pm \frac{16 \times 10^6}{785.4 \times 10^3}$ 20.372 N/mm

$\therefore$ Resultant stress at the edge, $B = f_0 + f_b = 12.732 - 20.372$

$= 33.104 \text{ N/mm}^2$ (compressive)

$\therefore$ Resultant stress at the edge, $A = f_0 - f_b = 12.732 - 20.372$

$= -7.640 \text{ N/mm}^2$ (tensile)

The stress distribution along the diameter is as shown in Figure 7.8(b).

Example 2

A short hollow cylindrical column carries a compressive force of 400 kN. The external diameter of the column is 200 mm and the internal diameter is 120 mm. Find the maximum permissible eccentricity of the load, if the allowable stresses are 60 N/mm^2 in compression and 25 N/mm^2 in tension.

Solution

External diameter, $D = 200$ mm

Internal diameter, $d = 120$ mm

Area of the section, $A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(200^2 - 120^2) = 2.01 \times 10^4 \text{ mm}^2$

Applied load, $P = 400 \text{ kN} = 4 \times 10^5 \text{ N}$

Direct stress, $f_0 = \frac{P}{A} = \frac{4 \times 10^5}{2.01 \times 10^4} = 19.9 \text{ N/mm}^2$ (compressive)

Let the eccentricity of the load = e mm.

Bending moment, $M = P \times e = (400 \times 10^5 \times e)$ N-mm

Section modulus, $z = \frac{\pi}{64}(D^4 - d^4) \times \left(\frac{2}{D}\right)$

$$= \frac{\pi}{32} \frac{(200^4 - 120^4)}{200} = 68.36 \times 10^4 \text{ mm}^3$$

Bending stress, $f_b = \pm \dfrac{M}{Z}$

$$= \pm \frac{(4 \times 10^5 \times e)}{(68.36 \times 10^4)}$$

$$= \pm\ 0.585 \times e$$

Resultant stress at extreme fibres $= f_0 \pm f_b = 19.90 \pm 0.585\ e$

$\therefore$ Maximum compressive stress $= (19.90 + 0.585\ e)$ . . . (i)

Minimum compressive stress $= (19.90 - 0.585\ e)$

or Maximum tensile stress $= (0.585\ e - 19.90)$ . . . (ii)

Thus, $(19.90 + 0.585\ e) \leq 60$ N/mm^2 (allowable compressive stress)

$\therefore$ $e \leq 68.55$ mm . . . (iii)

(iii) $(0.585\ e - 19.90) \leq 25$ N/mm^2 (allowable tensile stress)

$e \leq 8.72$ mm . . . (iv)

$\therefore$

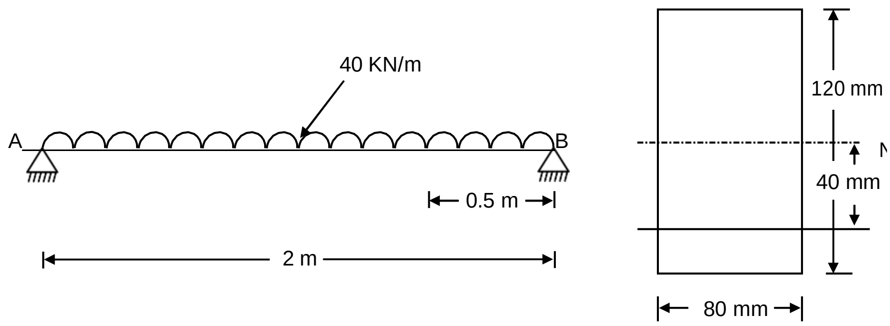
From these two conditions, the allowable maximum eccentricity = 8.72 mm from the centre of the section.

Example .3

A beam of rectangular section of 80 mm to 120 mm carries a uniformly distributed load of 40 kN/m over a span of 2 m an axial compressive force of 10 kN. Calculate

(a) maximum fibre stress,

(b) fibre stress at a point 0.50 m from the left end of the beam and 40 mm below the neutral axis.



(a) Loading **(b) Cross-section**

Figure 7.9

Solution

Bending moment, $M = \dfrac{w \times e^2}{8} = \dfrac{40 \times 2^2}{8} = 20$ kN-m $= 20 \times 10^6$ N-mm

Section modulus, $Z = \frac{1}{6} \times 80 \times (120)^2 = 1.92 \times 10^5 \text{ mm}^3$

Moment of inertial, $I = \frac{1}{12} \times (80) \times (120)^3 = 11.52 \times 10^6 \text{ mm}^4$

Axial load, $P = 10 \text{ kN} = 10 \times 10^3 \text{ N}$

Direct stress, $f_0 = \frac{P}{A} = \frac{10 \times 10^3}{(80 \times 120)} = 1.04 \text{ N/mm}^2$

Bending stress, $f_b = \pm \frac{M}{Z} = \frac{20 \times 10^6}{1.92 \times 10^5} = \pm 104.16 \text{ N/mm}^2$

∴ Maximum fibre stress $= 1.04 + 104.16 = 105.20 \text{ N/mm}^2$ (compressive)

∴ Bending moment at 0.50 m from left end,

$$M = \left(-40 \times 0.50 + 40 \times \frac{0.50^2}{2} \right)$$

$= -15$ kN-m

$= 15 \times 10^6$ N-mm (sagging)

∴ Bending stress at 40 mm below the neutral axis,

$$= \frac{M}{I} \cdot y$$

$$= \frac{15 \times 10^6}{(11.52 \times 10^6)} \times (-40)$$

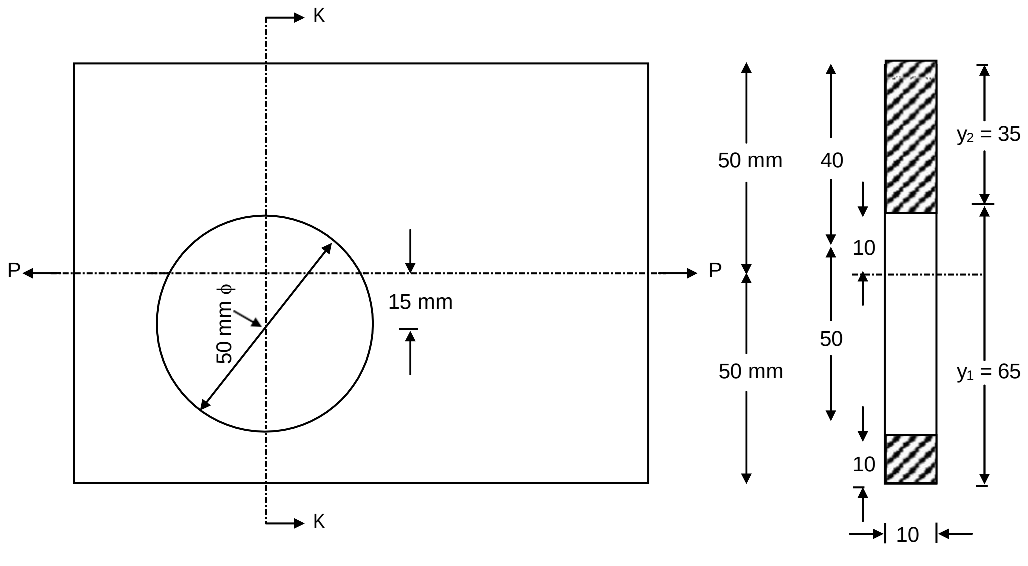
$= -52.08 \text{ N/mm}^2$ (tensile)

∴ Resultant fibre stress $= 1.04 - 52.08$

$= -51.04 \text{ N/mm}^2$ (tensile)

Example .4

A rectangular plate 10 mm thick with a hole of 50 mm diameter drilled on it is as shown in Figure 7.10. It is subjected to axial pull of 45 kN. Determine the greatest and the least intensities of stress at the critical cross-section of the plate.



Critical Section along

Solution

Area of section at the weakest point, $A = (10 \times 10) + (40 \times 10) = 500 \text{ mm}^2$.

To locate the centroidal axes, taking moments about *AB*,

$$y_1 = \frac{(10 \times 10) + (40 \times 10 \times 80)}{(10 \times 10) + 40 \times 10)} = 65 \text{ mm bottom}$$

Thus, $y_2 = 100 - 65 = 35$ mm

Moment of inertia about *xx*-axis,

$$I_{xx} = \frac{10 \times 40^3}{12} + 40 \times 10 \times (35 - 20)^2 + \frac{10 \times 10^3}{12} + 10 \times 10 \times (65 - 5)^2$$

$$= 50.42 \times 10^4 \text{ mm}^4$$

Axial load, $P = 45$ kN $= 45 \times 10^3$ N (tensile)

Direct stress, $f_0 = \frac{P}{A} = \frac{45 \times 10^3}{500} = 90 \text{ N/mm}^2$ (tensile)

Eccentricity, $e = 65 - 50 = 15$ mm

Bending stress along edge, $AB = \frac{P \times e}{I_{xx}} \times y$

$$= \left(\frac{45 \times 10^3 \times 15}{50.42 \times 10^4}\right) \times (-65)$$

$= -87 \text{ N/mm}^2$ (compressive)

Bending stress along edge, $CD = \left(\frac{45 \times 10^3 \times 15}{50.42 \times 10^4}\right) \times (35)$

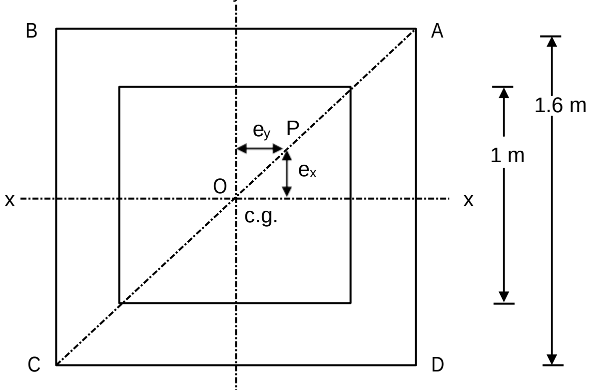
$= 46.8 \text{ N/mm}^2$ (compressive)

Maximum stress along edge, $AB = -(90 + 87) = -177 \text{ N/mm}^2$ (tensile)

Minimum stress along edge, $CD = -90 + 46.8 = -43.2 \text{ N/mm}^2$ (tensile).

Example .5

A short hollow pier 1.6 m × 1.6 m outsides and 1.0 m × 1.0 m intersides supports a vertical load of 2000 kN at a point located on a diagonal 0.5 m from the vertical axis of the pier. Calculate the normal stresses at the 4 corners of the section of the pier, neglecting its self weight.

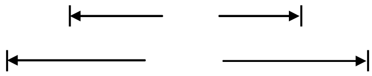


y

1

m

1.6
m



Solution

Figure 7.11 shows the section of the pier. At *P*, the load of 2000 kN is applied on the pier.

Area of cross-section $A = 1.6^2 - 1.0^2 = 1.56\ m^2$.

Section modulus, $Z_{xx} = Z_{yy} = \frac{1.60^4 - 1.00^4}{12} \times \left(\frac{2}{1.60}\right) = 0.5875\ m^3$

Eccentricity about *XX*-axis = Eccentricity about *YY*-axis

$= 0.50 \sin 45^o$

$= 0.353$ m

Bending moment about *XX*-axis = Bending moment about *YY*-axis

$= (2000 \times 0.353)$

$= 706$ kNm

Direct stress, $f_0 = \frac{P}{A} = \frac{2000}{1.56} = 1282.05\ kN/m^2$ (compressive)

Bending stress about *XX*-axis = Bending stress about *YY*-axis

$= \pm \frac{M}{Z} = \pm \frac{706}{0.5785} = \pm 1220.4\ kN/m^2$

$\therefore$ Resultant stresses at corners, $f = \frac{P}{A} \pm \frac{M_{xx}}{Z_{xx}} \pm \frac{M_{yy}}{Z_{yy}}$

Stress at corner $A = 1282.05 + 1220.4 + 1220.4$

$= 3722.85\ kN/m^2$ (compressive)

Stress at corner $B = 1282.05 - 1220.4 + 1220.4$

$= 1282.05\ kN/m^2$ (compressive)

Stress at corner $C = 1282.05 - 1220.4 - 1220.4$

$= -1158.75\ kN/m^2$ (tensile)

Stress at corner $D = 1282.05 + 1220.4 - 1220.4$

$= 1282.05\ kN/m^2$ (compressive)

SAQ 1

(a) A cast iron column of rectangular section 200 mm × 300 mm carries a vertical load of 250 kN at a point 30 mm away from the c.g. of the section on a line passing through the centroid and parallel to the longer side. Determine the maximum stress at the edges of the line passing through the centroid on which the point of application of load lies.

(b) If the above member is subjected to a compressive load of 100 kN acting at a point 40 mm away from its c.g. and along a diagonal, what will be the resultant stresses at the four corners of the top face of the column?

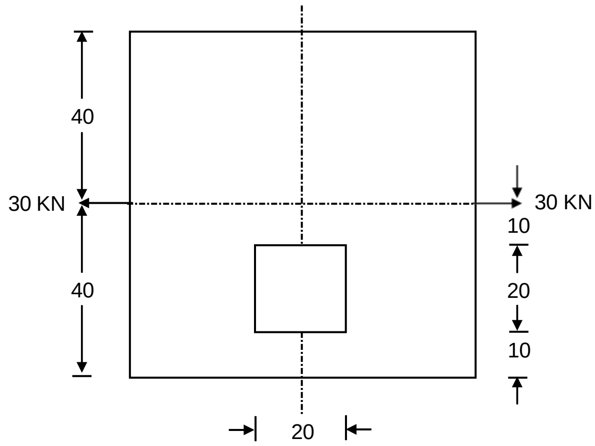
(c) A short column of hollow circular section of internal diameter '*d*' and external diameter '*D*' is loaded with a compressive load W. Determine the

m a xi m u m di st a n c e

of the point of application of the load from the centre of the section, such that the tensile stress does not exist at any point of the cross-section, if $D = 1.5\,d$.

SAQ 2

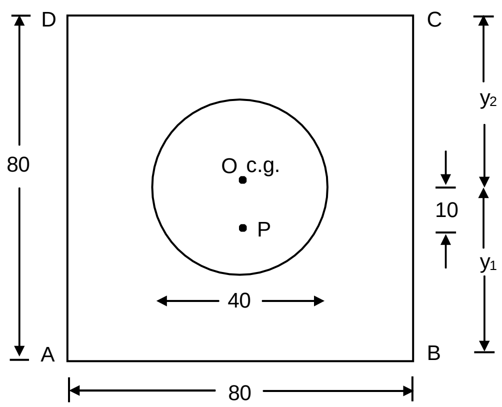
(a) A rectangular plate 20 mm thick, containing a square hole of 20 mm side as shown in Figure 7.12 is subjected to an axial pull of 30 kN. Determine the greatest and least tensile stresses at the critical section of the plate.



(b) The cross-section of a short column is shown in Figure 7.13. A vertical load *W* kN acts at the point *P*.

(i) Determine the value of *W* if the maximum stress set up in the cross-section is not to exceed 75 N/mm^2.

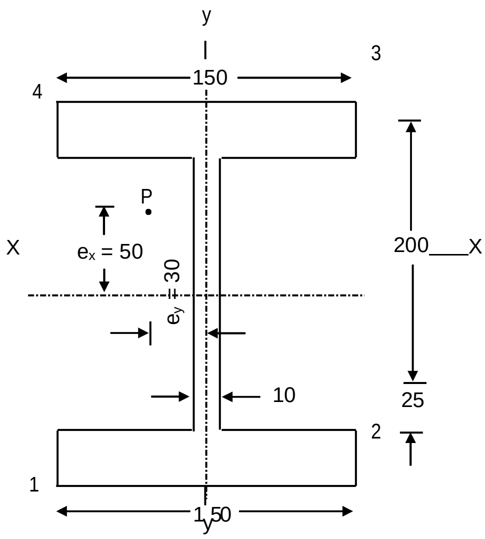
(ii) Draw the stress distribution diagram along the edge *AD*.



(c) A short hollow cylindrical cast iron column having outside diameter 300 mm and inside diameter 200 mm was cast in a factory. On inspection it was found that the bore is eccentric in such a way that the thickness varies from 30 mm at one end to 70 mm at the outer. Calculate the extreme intensities of stress induced in the section, if column carries a load of 800 kN along the axis of the bore.

Example .6

A rolled steel I-section, flanges 150 mm wide and 25 mm thick, web 200 mm long and 10 mm thick (Figure 7.14) is used to carry and axial load of 800 kN. The load line is eccentric, 50 mm above XX and 30 mm to the left of yy. Find the maximum and minimum stress intensities in the section.



Solution

$$A = 2 \times (150 \times 25) + (200 \times 10)$$

Area of the cross-section, $= 9500 \text{ mm}^2$

Moment of inertia about XX-axis,

$$I_{xx} = \frac{150 \times 250^3}{12} - \frac{140\ (20)^3}{12} = 8565 \times 10^4 \text{ mm}^4$$

Moment of inertia about YY-axis,

$$I_{yy} = \frac{2 \times (25 \times 150^3)}{12} - \frac{200\ (10)^3}{12} = 1407 \times 10^4 \text{ mm}^4$$

Eccentricity, $e_x = 50$ mm

$e_y = -30$ mm

Vertical load, $W = 800$ kN

Direct stress at any point, $f_0 = \frac{P}{A} = \frac{800 \times 10^3}{9500} = 84.2 \text{ N/mm}^2$

Maximum bending compressive stress will occur at edge 4 of the section in Figure 7.14.

$$(f_b)_4 = \left(\frac{P \times e_x}{I_{xx}}\right) \times y + \left(\frac{P \times e_y}{I_{yy}}\right) \times x$$

$$= \left(\frac{800 \times 10^3 \times 50}{8565 \times 10^4}\right) \times (-125) + \left(\frac{800 \times 10^3 \times (-30)}{1407 \times 10^4}\right) \times 75$$

$$= 186.4 \text{ N/mm}^2$$

Maximum bending tensile stress will occur at edge 2 at the section.

$$(f_b)_2 = \left(\frac{800 \times 10^3 \times 50}{8565 \times 10^4}\right) \times (-125) + \left(\frac{800 \times 10^3 \times (-30)}{1407 \times 10^4}\right) \times 75$$

$= -18.64$ N/mm^2

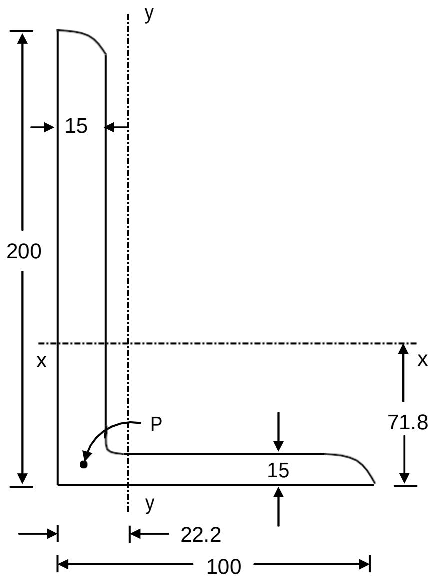
Resultant stress in the section would be as follows :

Maximum at corner 4 = 84.2 + 186.4 = 270.6 N/mm^2

Minimum at corner 5 = 84.2 – 186.4 = – 102.2 N/mm^2 (tensile)

Example 7

A short piece of ISA (200 × 100 × 15) angle carries a compressive load, the line of action of which coincides with the intersection of the middle planes of the legs. If the maximum compressive stress is not to exceed 112 N/mm^2, what is the safe axial load *P*? Given $A = 4278$ mm^2, $r_{xx} = 64$ mm, $r_{yy} = 26.4$ mm.



Solution

Area of cross-section $A = 4278$ mm^2

Eccentricity of load with respect of *xx*-axis = (71.8 – 7.5) = 64.3 mm

Eccentricity of load with respect to *yy*-axis = 22.2 – 7.5 = 14.7 mm

Maximum compressive stress at any section

$$= \frac{P}{A} + \frac{M_{xx}}{I_{xx}} \times y + \frac{M_{yy}}{I_{yy}} \times x$$

or $$f_{max} = \frac{P}{A}\left(1 + \frac{e_{xx}}{r_{xx}^2} \times y + \frac{e_{yy}}{r_{yy}^2} \times x\right)$$

Here, $r_{xx} = 64$ mm and $r_{yy} = 26.4$ mm

$f_{max} = 112$ N/mm^2

$$\therefore \quad 112 = \frac{P}{A}\left(1 + \frac{64.3 \times 71.8}{(64)^2} + \frac{14.7 \times 22.2}{(26.4)^2}\right)$$

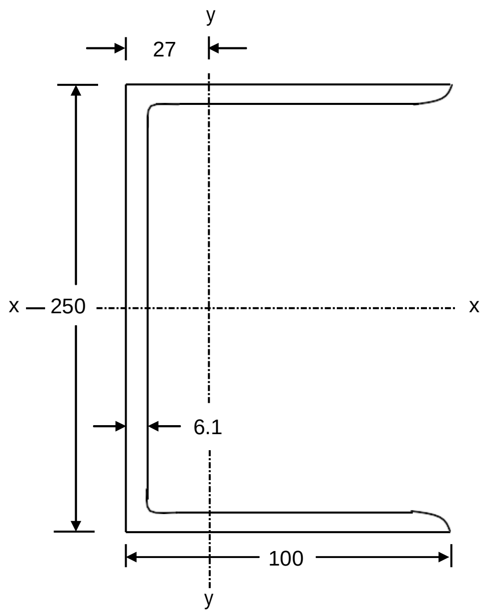
$$= \frac{P}{4278}\,[1 + 1.127 + 0.4684]$$

$\therefore \quad P = 184.6$ kN.

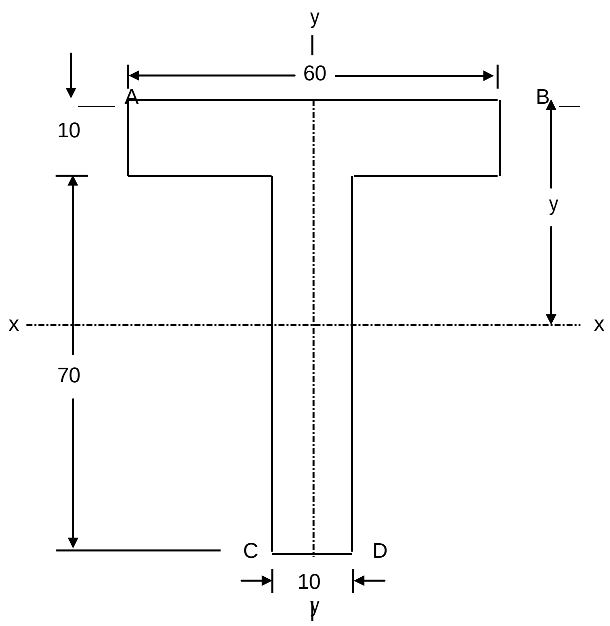
SAQ 3

(a) A short piece of ISLC 250 channel (Figure 7.16) carries a compressive load, the line of action of which through the centroid of the web. If the allowable maximum compressive stress is 112 N/mm^2, calculate the safe axial load.

(Given : A = 3565 mm^2, r_{yy} = 28.9 mm, t = 6.1 mm).



(b) A bar of T-section as shown in Figure 7.17 is subjected to a longitudinal pull P applied at a point on the yy-axis but not at the centroid of the section. Determine the magnitude of P and the position of its line of section if the stresses across the section vary from 10 N/mm^2 compression at the top to 120 N/mm^2 tension at the bottom.

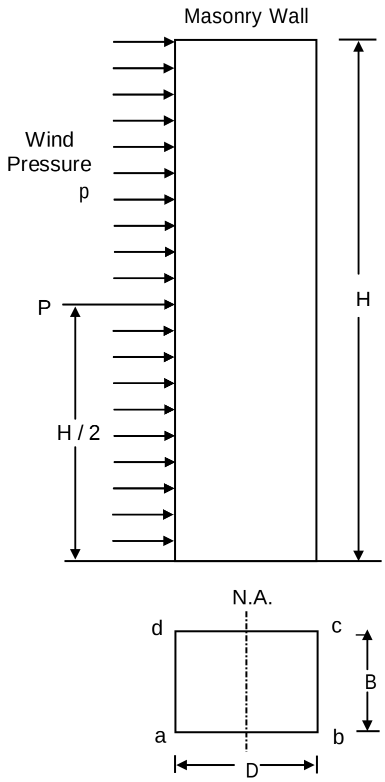


WALLS AND PILLARS

Wind Forces on Walls and Pillars

Many a times masonry walls and chimney shafts are subjected to strong wind pressures. The weight of the walls or the chimney produces compressive stress in the base while the wind pressure introduces bending moment producing tensile and compressive stresses in the base.

Figure shows a masonry wall of height *H* and rectangular section $B \times D$. the horizontal wind pressure of intensity '*p*' is acting on the face of width *B*.



Let the unit

weight of the material of the structure = γSelf weight

of the structure $W = \gamma\,(BDH)$

Area of cross-section at the base = $B \times D$

Compressive stress due to self weight of the structure on its base,

$$f_0 = \frac{\gamma\,(BDH)}{BD} = \gamma H$$

Total wind force on the vertical face = $P = p \times BH$

Distance of centre of gravity of the wind force from the base $= \frac{H}{2}$

Bending moment, $M = \frac{PH}{2} = p \times \frac{BH^2}{2}$

Section modulus, $Z = \frac{BD^2}{6}$

Bending stress, $f_b = \frac{M}{Z}$ $\frac{BH^2}{}$ $\frac{6}{}$ $\frac{H^2}{}$

$= \pm$
$p \times$

$2 \quad \times \quad BD^2 \qquad = \pm\, 3p$
$\times$
D^2

Due to Bending Moment (BM), there will be maximum tensile stress along edge '*ad*' and maximum compressive stress along edge '*bc*' of the base.

Resultant stress, $$f = f_0 - 3p\frac{H^2}{D^2} \text{ along edge } ad$$

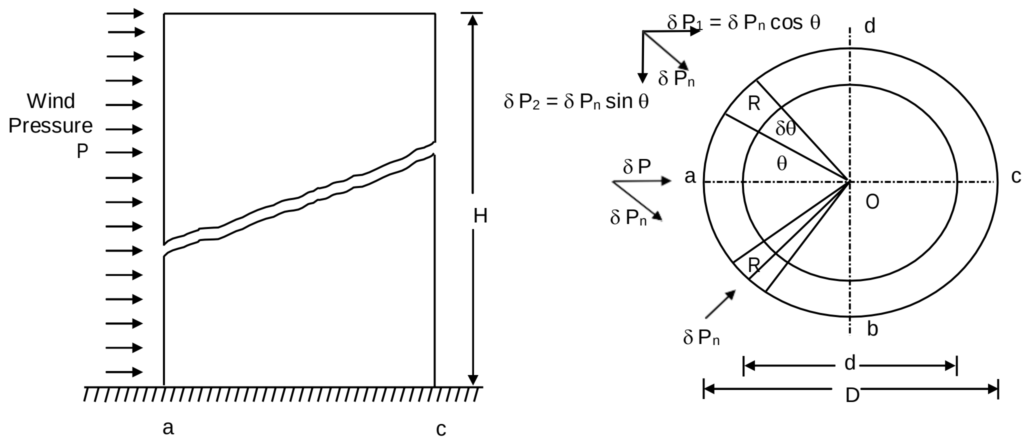
$$f = f_0 + 3p\frac{H^2}{D^2} \text{ along edge } bc.$$

Wind Forces on Chimneys

Having seen the stress distribution for a wall subjected to wind forces, let us consider the effect of wind forces on all chimneys.

Figure 7.19 shows a cylindrical chimney of height *H*, external diameters *D*, internal diameter *d*, subjected to horizontal wind pressure *p* as shown.

If γ is the unit weight of the masonry structure, direct stress due to the weight of the structure on its base $f_0 = \gamma H$.



(a) Elevation (b) Cross-section

Consider a small strip of width $R\,\delta\theta$, subtending an angle $\delta\theta$ at the centre and making anangle θ with the axis *ac* of the section,

$$\delta P = \text{Wind force reaching the small strip}$$

$$= p \times R\,\delta\theta \times H \cos\theta$$

$$= p\,H\,R\,\delta\theta \cos\theta$$

Component of the force normal to the strip,

$$\delta P_n = \delta P \cos\theta$$

$$= p\,H\,R \cos\theta \times \delta\theta \times \cos\theta$$

$$= p\,H\,R \cos^2\theta\,\delta\theta$$

Horizontal component of

$$\delta P_n = \delta P_1 = \delta P_n \cos\theta$$

$$= p\,H\,R \cos^3\theta\,\delta\theta$$

Another horizontal component of

$$\delta P_n = \delta P_2 = \delta P_n \sin\theta$$

While summing up, this component gets cancelled when we consider a strip in the other quadrant as shown in Figure 7.19, while the components of $P_n \cos \theta$ are added up.

$\therefore$ Total force in the direction X-X $= 2\delta P_n \cos \theta$

$$= 2 p\ H R \cos^3 \theta\ \delta\theta$$

Integrating over the whole exposed surface, from $\theta = 0^o$ to 90^o.

$$\text{Total wind force } P = \int_0^{\frac{\pi}{2}} 2 p\ H\ R \cos^3 \theta\ \delta\theta$$

$$= p\ DH \times \frac{2}{3} = k \times p\ DH$$

where, k = Coefficient of wind resistance, and

DH = Projected area of the curved surface.

$$\text{BM due to wind force, } M = \frac{PH}{2}$$

$$= p\ DH\ \frac{2}{3} \times \frac{H}{2}$$

$$= \frac{p\ DH^2}{3}$$

$$\text{Section modulus, } Z = \frac{\pi\,(D^4 - d^4)}{32 D}$$

$$\text{Bending stress, } f_b = \pm \frac{M}{Z}$$

Once you calculate the bending stress, the extreme fibre stresses can be obtained by summing up with the direct stress due to self weight.

Example .8

A 10 m high masonry chimney wall of rectangular section 4 m $\times$ 1.5 m is subjected to a horizontal wind pressure of 1500 N/m^2 on the 4 m side. Find the maximum and minimum stress intensities induced on the base. Take unit weight of masonry as 22 kN/m^3.

Solution

Breadth, B = 4 m; Height, H = 10 m; and Depth, D = 1.5 m

Cross-sectional area at the base, $A = 4 \times 1.5 = 6$ m^2

Self weight of the wall, $W = \gamma BDH$

$$= 22 \times (4 \times 1.5 \times 10) = 1320 \text{ kN}$$

Direct compressive stress at the base, $f_0 = \frac{W}{A} = \frac{1320}{6} = 220$ kN/m^2

Wind pressure, = 1.5 kN/m^2

Wind force on the vertical face of side 4 m, $P = p \times B \times H$

$$= 1.5 \times 4 \times 10 = 60 \text{ kN}$$

Distance of centre of gravity of P from base, $= \frac{H}{2}$ = 5 m

Bending moment, $M = \frac{PH}{2} = 60 \times 5 = 300$ kNm

Section modulus, $Z = \frac{BD^2}{6} = \frac{4 \times 1.5^2}{6} = 1.5\ m^3$

Bending stress due to bending moment, $= f_b = \pm \frac{M}{Z}$

$$= \pm \frac{300}{1.5} = \pm\ 200\ kN/m^2$$

Maximum stress induced $= 220 + 200 = 420\ kN/m^2$ (compressive)

Minimum stress induced $= 220 - 200 = 20\ kN/m^2$ (compressive)

Example .9

A masonry chimney 20 m high of uniform circular section, 5 m external diameter and 3 m internal diameter has to withstand a horizontal wind pressure of intensity 2 kN/m^2 of the projected area. Find the maximum and minimum stress intensities at the base. Take unit weight of masonry as 21 kN/m^3.

Solution

Height of the chimney, H = 20 m

External diameter, D = 5 m

Internal diameter, d = 3 m

Unit weight of masonry, $\gamma = 21\ kN/m^3$

Direct compressive stress due to self weight on the base of the chimney,

$$f_0 = \gamma H = (21 \times 20) = 420\ kN/m^2$$

Wind pressure, $p = 2\ kN/m^2$

Projected area, $A = DH = 5 \times 20 = 100\ m^2$

Wind force, $P = pA = 2 \times 100 = 200$ kN

Distance of centre of gravity of the wind force from base, $= \frac{H}{2} = 10$ m

Bending moment, $M = \frac{PH}{2} = 200 \times 10 = 2000$ kNm

Section modulus, $Z = \frac{\pi}{32} \times \frac{(D^4 - d^4)}{D}$

$$= \frac{\pi}{32} \frac{(5^4 - 3^4)}{5} = 10.68\ m^3$$

Bending stress, $f_b = \pm \frac{M}{Z} = \pm \frac{2000}{10.68}$

$$= \pm\ 187.266\ kN/m^2$$

Maximum stress induced $= 420 + 187.266 = 607.266\ kN/m^2$

Minimum stress induced $= 420 - 187.266 = 232.734\ kN/m^2$.

SAQ 4

(a) The section of a masonry pier is a hollow rectangle, external dimensions 4 m × 1.2 m and internal dimensions 2.4 m × 0.6 m. a horizontal thrust of 30 kN is exerted at the top of the pier in the vertical plane bisecting the length 4 m. the height of the pier is 5 m and unit weight of masonry is 22.5 kN/m^3. Calculate the maximum and minimum intensities of stress at the base.

(b) A cylinder chimney of a hollow circular section, 2 m external diameter and 1 m internal diameter is 25 m high. Given that the horizontal intensity of wind pressure is 1 kN/m^3, determine the extreme intensities of stress at the base. Take coefficient of wind resistance as 0.6 and unit weight of masonry as 22.8 kN/m^3.

(c) A tapering chimney of hollow circular section is 45 m high. Its external diameter at the base is 3.6 m and at the top it is 2.4 m. It is subjected to wind pressure of 2.2 kN/m^2 of the projected area. Calculate the overturning moment at the base. If the weight of the chimney is 6000 kN and the internal diameter at the base is 1.2 m, determine the maximum and minimum stress intensities at the base.

SUMMARY

Let us conclude the unit by summarising what we have covered in it. In this unit, we have

- defined neutral axis,
- introduced the effect of axial force and bending moment on different sections,
- obtained the core or kernel of different sections,
- calculated the stress distribution on different cross-sections due to axial force and bending moment, and
- obtained stress distribution on walls and pillars due to wind forces.

ANSWERS TO SAQs

SAQ 1

(a) 6.67 N/mm^2 (compression) and 1.67 N/mm^2 (compression).

(b) Eccentricity : e_x = 33.28 mm, e_y = 22.18 mm; and

Stresses : 1.67 N/mm^2, 3.88 N/mm^2, 1.67 N/mm^2 and – 0.552 N/mm^2.

(c) $e = \frac{13}{72} D$

SAQ 2

(a) 38.806 N/mm^2 (greatest) and 9.181 N/mm^2 (least).

(b) (i) W = 237.2 kN.

(ii) Stress variations : at A = 17.4 N/mm^2 and at B = 75 N/mm^2.

¯(c) Centre of gravity from one end,

$x =$ 133.8 mm

ricity, e = 36.2 mm and I_{yy} = 296.65 × 10^6 mm^4

Maximum stress = (20.36 + 16.23) = 36.59 N/mm^2 (compressive)

Minimum stress = (20.36 – 13.05) = 7.31 N/mm^2 (compressive)

SAQ 3

Eccent

(a) 225 kN.

(b) From two equations involving P and e for extreme fibre stresses at op and bottom

$$P = 43.210 \text{ kN, and } e = 28.46$$

SAQ 4

Maximum : 281.42 kN/mm^2 and Minimum : – 56.42 kN/m^2

(a) 1079.3 kN/m^2 and 60.70 kN/m^2 (compressive)

(b) Moment : 6237 kN m

Stresses : 2051.7 kN/m^2 (compressive) and 705.5 kN/m^2 (tensile).

UNIT -4

THIN CYLINDERS AND THICK CYLINDERS

UNIT-IV

THIN CYLINDERS AND THICK CYLINDERS

Members Subjected to Axisymmetric Loads

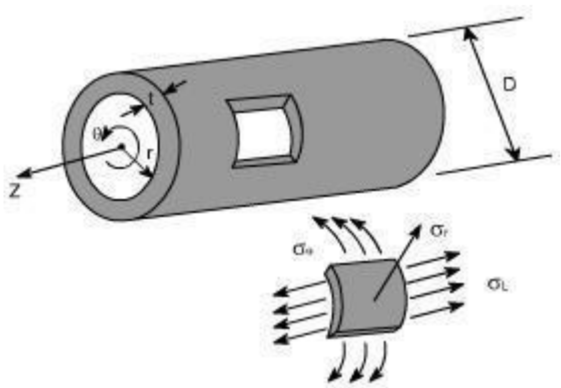
Thin walled cylinder:

Preamble : Pressure vessels are exceedingly important in industry. Normally two types of pressure vessel are used in common practice such as cylindrical pressure vessel and spherical pressure vessel.

In the analysis of this walled cylinders subjected to internal pressures it is assumed that the radial plans remains radial and the wall thickness dose not change due to internal pressure. Although the internal pressure acting on the wall causes a local compressive stresses (equal to pressure) but its value is neglibly small as compared to other stresses & hence the sate of stress of an element of a thin walled pressure is considered a biaxial one.

Further in the analysis of them walled cylinders, the weight of the fluid is considered neglible.

Let us consider a long cylinder of circular cross - section with an internal radius of R $_2$ and a constant wall thickness't' as showing fig.



This cylinder is subjected to a difference of hydrostatic pressure of 'p' between its inner and outer surfaces. In many cases, 'p' between gage pressure within the cylinder, taking outside pressure to be ambient.

By thin walled cylinder we mean that the thickness't' is very much smaller than the radius R_i and we may quantify this by stating than the ratio t / R_i of thickness of radius should be less than 0.1.

An appropriate co-ordinate system to be used to describe such a system is the cylindrical polar one r, □ , z shown, where z axis lies along the axis of the cylinder, r is radial to it and □□is the angular co-ordinate about the axis.

The small piece of the cylinder wall is shown in isolation, and stresses in respective direction have also been shown.

Such a component fails in since when subjected to an excessively high internal pressure. While it might fail by bursting along a path following the circumference of the cylinder. Under normal circumstance it fails by circumstances it fails by bursting along a path parallel to the axis. This suggests that the hoop stress is significantly higher than the axial stress.

In order to derive the expressions for various stresses we make following
APPLICATIONS:

Liquid storage tanks and containers, water pipes, boilers, submarine hulls, and certain air plane components are common examples of thin walled cylinders and spheres, roof domes.

ANALYSIS : In order to analyse the thin walled cylinders, let us make the following assumptions :

- There are no shear stresses acting in the wall.
- The longitudinal and hoop stresses do not vary through the wall.
- Radial stresses σ_r which acts normal to the curved plane of the isolated element are neglibly small as compared to other two stresses especially when $\left[\frac{t}{R_i} < \frac{1}{20}\right]$

The state of tress for an element of a thin walled pressure vessel is considered to be biaxial, although the internal pressure acting normal to the wall causes a local compressive stress equal to the internal pressure, Actually a state of tri-axial stress exists on the inside of the vessel. However, for then walled pressure vessel the third stress is much smaller than the other two stresses and for this reason in can be neglected.

Thin Cylinders Subjected to Internal Pressure:

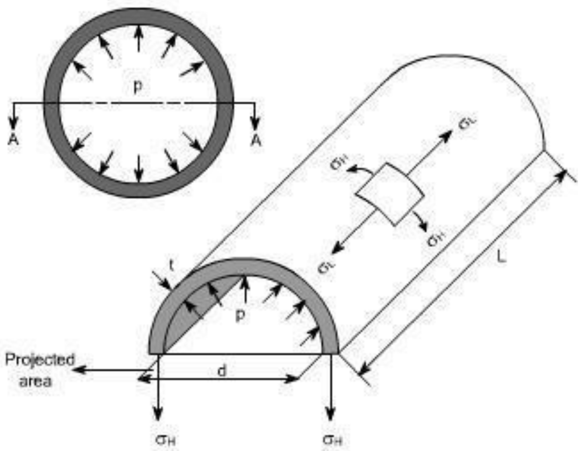
When a thin – walled cylinder is subjected to internal pressure, three mutually perpendicular principal stresses will be set up in the cylinder materials, namely

- Circumferential or hoop stress
- The radial stress
- Longitudinal stress

now let us define these stresses and determine the expressions for them

circumferential stress:

This is the stress which is set up in resisting the bursting effect of the applied pressure and can be most conveniently treated by considering the equilibrium of the cylinder.



In the figure we have shown a one half of the cylinder. This cylinder is subjected to an internal pressure p.

i.e. p = internal pressure d

= inside diameter

L = Length of the cylinder t

= thickness of the wall

Total force on one half of the cylinder owing to the internal pressure 'p'

= p x Projected Area

= p x d x L

= p .d. L ------------------------ (1)

The total resisting force owing to hoop stresses σ_H set up in the cylinder walls

= 2 .σ_H .L.t -------------------- (2)

Because σ_H.L.t. is the force in the one wall of the half cylinder. the

equations (1) & (2) we get

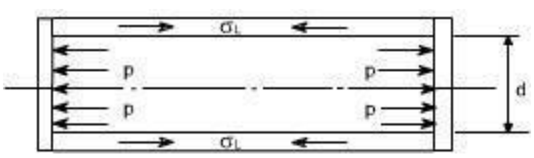
2 . σ_H . L . t = p . d . L

σ_H = (p . d) / 2t

Circumferential or hoop Stress (σ_H) = (p .d)/ 2t

ngitudinal Stress:

Consider now again the same figure and the vessel could be considered to have closed ends and contains a fluid under a gage pressure p. Then the walls of the cylinder will have a longitudinal stress as well as a circumferential stress.

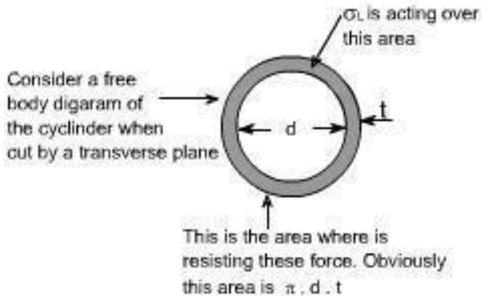


Total force on the end of the cylinder owing to internal pressure

= pressure x area

= p x $\pi d^2/4$

Area of metal resisting this force = πd.t. (approximately) because



Hence the longitudnal stresses

$$\text{Set up} = \frac{\text{force}}{\text{area}} = \frac{[p \times \pi d^2/4]}{\pi d t}$$

$$= \frac{pd}{4t} \quad \text{or} \quad \sigma_L = \frac{pd}{4t}$$

or alternatively from equilibrium conditions

$$\sigma_L.(\pi d t) = p.\frac{\pi d^2}{4}$$

$$\text{Thus } \boxed{\sigma_L = \frac{pd}{4t}}$$

Change in Dimensions :

The change in length of the cylinder may be determined from the longitudinal strain.

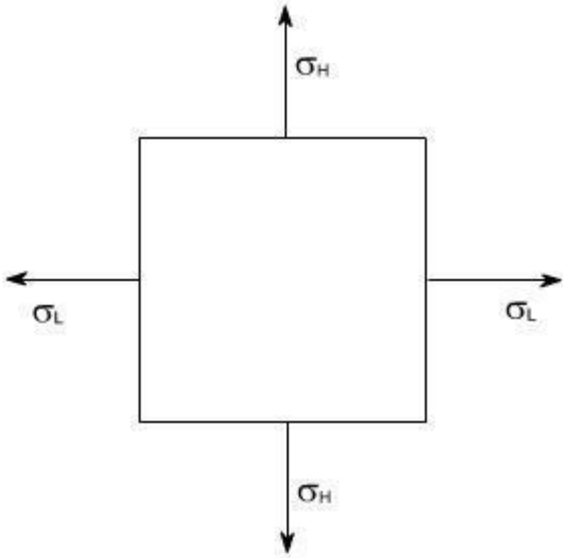
Since whenever the cylinder will elongate in axial direction or longitudinal direction, this will also get decreased in diameter or the lateral strain will also take place. Therefore we will have to also take into consideration the lateral strain.as we know that the poisson's ratio (ν) is

$$\nu = \frac{-\text{lateral strain}}{\text{longitudnal strain}}$$

where the -ve sign emphasized that the change is negative

Consider an element of cylinder wall which is subjected to two mutually σ^r normal stresses σ_L and σ_H
.

Let E = Young's modulus of elasticity


σH
σL
σL
σH

Resultant Strain in longitudnal direction $= \frac{\sigma_L}{E} - \nu\frac{\sigma_H}{E} = \frac{1}{E}(\sigma_L - \nu\sigma_H)$

recalling

$$\sigma_L = \frac{pd}{4t} \qquad \sigma_H = \frac{pd}{2t}$$

$\in_1$ (longitudnal strain) $= \frac{pd}{4Et}[1-2\nu]$

or

Change in Length = Longitudalstrain x original Length

$= \in_1 . L$

Similarly the hoop Strain $\in_2 = \frac{1}{E}(\sigma_H - \nu\sigma_L) = \frac{1}{E}\left[\frac{pd}{2t} - \nu\frac{pd}{4t}\right]$

$$\in_2 = \frac{pd}{4Et}[2-\nu]$$

Infact $\in_2$ is the hoop strain if we just go by the definition then

$$\in_2 = \frac{\text{Change in diametre}}{\text{Original diametre}} = \frac{\delta d}{d}$$

where d = original diameter.

if we are interested to find out the change in diametre then

Change in diametre $= \in_2$ .Original diametre

i.e $\delta d = \in_2 .d$ substituting the value of $\in_2$ we get

$$\delta d = \frac{p.d}{4.t.E}[2-\nu].d$$

$$= \frac{p.d^2}{4.t.E}[2-\nu]$$

i.e $\boxed{\delta d = \frac{p.d^2}{4.t.E}[2-\nu]}$

umetric Strain or Change in the Internal Volume:

When the thin cylinder is subjected to the internal pressure as we have already calculated that there is a change in the cylinder dimensions i.e, longitudinal strain and hoop strains come into picture. As a result of which there will be change in capacity of the cylinder or there is a change in the volume of the cylinder hence it becomes imperative to determine the change in volume or the volumetric strain.

The capacity of a cylinder is defined as =

Area X Length

$= \pi d^2/4 \times L$

Let there be a change in dimensions occurs, when the thin cylinder is subjected to an internal pressure.

(i) The diameter **d** changes to **δ d + δ d**
ii)The length **L** changes to **δ L + δ L**

Therefore, the change in volume = Final volume - Original volume

$$= \frac{\pi}{4}[d+\delta d]^2.(L+\delta L) - \frac{\pi}{4}d^2.L$$

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\frac{\pi}{4}[d+\delta d]^2.(L+\delta L) - \frac{\pi}{4}d^2.L}{\frac{\pi}{4}d^2.L}$$

$$\in_v = \frac{\left\{[d+\delta d]^2.(L+\delta L) - d^2.L\right\}}{d^2.L} = \frac{\left\{(d^2+\delta d^2+2d.\delta d).(L+\delta L) - d^2.L\right\}}{d^2.L}$$

simplifying and neglecting the products and squares of small quantities, i.e. δd & δL hence

$$= \frac{2d.\delta d.L + \delta L.d^2}{d^2 L} = \frac{\delta L}{L} + 2.\frac{\delta d}{d}$$

By definition $\frac{\delta L}{L}$ = Longitudnal strain

$\frac{\delta d}{d}$ = hoop strain, Thus

Volumetric strain = longitudnal strain + 2 x hoop strain

on substituting the value of longitudnal and hoop strains we get

$$\in_1 = \frac{pd}{4tE}[1-2\nu] \quad \& \quad \in_2 = \frac{pd}{4tE}[1-2\nu]$$

$$\text{or Volumetric} = \in_1 + 2\in_2 = \frac{pd}{4tE}[1-2\nu] + 2.\left(\frac{pd}{4tE}[1-2\nu]\right)$$

$$= \frac{pd}{4tE}\{1-2\nu+4-2\nu\} = \frac{pd}{4tE}[5-4\nu]$$

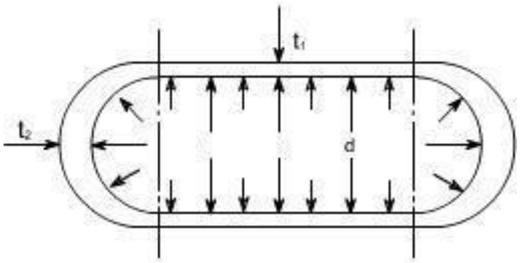
$$\text{Volumetric Strain} = \frac{pd}{4tE}[5-4\nu] \quad \text{or} \quad \boxed{\in_v = \frac{pd}{4tE}[5-4\nu]}$$

Therefore to find but the increase in capacity or volume, multiply the volumetric strain by original volume.

Hence

Change in Capacity / Volume or

$$\text{Increase in volume} = \frac{pd}{4tE}[5 - 4\nu]V$$



indrical Vessel with Hemispherical Ends:

Let us now consider the vessel with hemispherical ends. The wall thickness of the cylindrical and hemispherical portion is different. While the internal diameter of both the portions is assumed to be equal

Let the cylindrical vassal is subjected to an internal pressure p.

hoop or circumferential stress = σ_{HC} 'c' here synifies the cylindrical portion.

$$= \frac{pd}{2t_1}$$

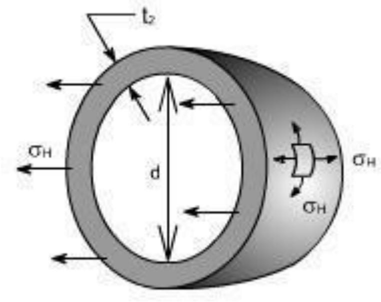
longitudnal stress = σ_{LC}

$$= \frac{pd}{4t_1}$$

hoop or circumferential strain $\in_2 = \frac{\sigma_{HC}}{E} - \nu\frac{\sigma_{LC}}{E} = \frac{pd}{4t_1E}[2-\nu]$

or $$\in_2 = \frac{pd}{4t_1E}[2-\nu]$$

For The Hemispherical Ends:



Because of the symmetry of the sphere the stresses set up owing to internal pressure will be two mutually perpendicular hoops or circumferential stresses of equal values. Again the radial stresses are neglected in comparison to the hoop stresses as with this cylinder having thickness to diameter less than1:20.

Consider the equilibrium of the half – sphere

Force on half-sphere owing to internal pressure = pressure x projected Area

$= p.\ \pi d^2/4$

Resisting force $= \sigma_H\ .\ \pi.d.t_2$

$\therefore \qquad p.\frac{\pi.d^2}{4} = \sigma_H\ .\ \pi d.t_2$

$\Rightarrow \ \sigma_H \text{(for sphere)} = \frac{pd}{4t_2}$

similarly the hoop strain $= \frac{1}{E}\left[\sigma_H - \nu.\sigma_H\right] = \frac{\sigma_H}{E}[1-\nu] = \frac{pd}{4t_2E}[1-\nu]$ or $\boxed{\epsilon_{2s} = \frac{\mathbf{pd}}{\mathbf{4t_2E}}[\mathbf{1-\nu}]}$

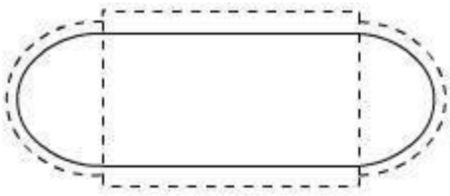


Fig – shown the (by way of dotted lines) the tendency, for the cylindrical portion and the spherical ends to expand by a different amount under the action of internal pressure. So owing to difference in stress, the two portions (i.e. cylindrical and spherical ends) expand by a different amount. This incompatibly of deformations causes a local bending and sheering stresses in the neighbour hood of the joint. Since there must be physical continuity between the ends and the cylindrical portion, for this reason, properly curved ends must be used for pressure vessels.

Thus equating the two strains in order that there shall be no distortion of the junction

$$\frac{pd}{4t_1E}[2-\nu] = \frac{pd}{4t_2E}[1-\nu] \quad \text{or} \quad \frac{t_2}{t_1} = \frac{1-\nu}{2-\nu}$$

But for general steel works $\nu = 0.3$, therefore, the thickness ratios becomes

$t_2 / t_1 = 0.7/1.7$ or

$t_1 = 2.4\ t_2$

i.e. the thickness of the cylinder walls must be approximately 2.4 times that of the hemispheroid ends for no distortion of the junction to occur.

SUMMARY OF THE RESULTS : Let us summarise the derived results

(A) The stresses set up in the walls of a thin cylinder owing to an internal pressure p are :

(i) Circumferential or loop stress

pd/2t

(ii) Longitudinal or axial stress

pd/4t

Where d is the internal diameter and t is the wall thickness of the cylinder. then

Longitudinal strain $e_L = 1 / E\ [\sigma_L - \nu \sigma_H]$

Hoop stain $e_H = 1 / E\ [\sigma_H - \nu \sigma_L]$

(B) Change of internal volume of cylinder under pressure

$$= \frac{pd}{4tE}[5-4\nu]V$$

(C) Fro thin spheres circumferential or loop stress

$$\sigma_H = \frac{pd}{4t}$$

Thin rotating ring or cylinder

Consider a thin ring or cylinder as shown in Fig below subjected to a radial internal pressure p caused by the centrifugal effect of its own mass when rotating. The centrifugal effect on a unit length of the circumference is

$p = m\,\omega^2\,r$

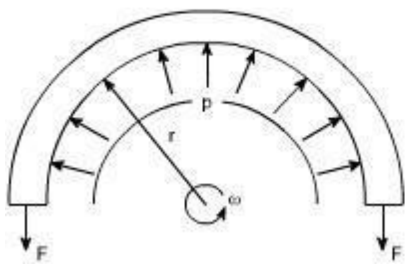


Fig 19.1: Thin ring rotating with constant angular velocity ω

Here the radial pressure 'p' is acting per unit length and is caused by the centrifugal effect if its own mass when rotating.

Thus considering the equilibrium of half the ring shown in the figure, 2F = p x 2r

(assuming unit length), as 2r is the projected area

F = pr

Where F is the hoop tension set up owing to rotation.

The cylinder wall is assumed to be so thin that the centrifugal effect can be assumed constant across the wall thickness.

F = mass x acceleration = $\mathbf{m\,\omega^2\,r \times r}$

This tension is transmitted through the complete circumference and therefore is resisted by the complete cross – sectional area.

Hoop stress = F/A = $\mathbf{m\,\omega^2\,r^2 / A}$

Where A is the cross – sectional area of the ring.

Now with unit length assumed m/A is the mass of the material per unit volume, i.e. the density ρ .

hoop stress = $\boldsymbol{\rho\,\omega^2\,r^2\;\sigma_H}$

$\boldsymbol{=\rho.\,\omega^2\,.\,r^2}$

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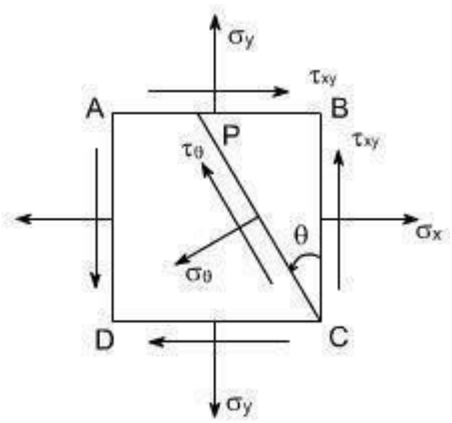
UNIV-V

UNSYMMETRICAL BENDING AND SHEAR CENTRE

GRAPHICAL SOLUTION MOHR'S STRESS CIRCLE

The transformation equations for plane stress can be represented in a graphical form known as Mohr'scircle. This grapical representation is very useful in depending the relationships between nor mal and shear stresses acting on any inclined plan e at a point in a stresses body.

To draw a Mohr's stress circle consider a complex stress system as shown in the figure



The above system represents a co mplete stress system for any condition of applied load in t wo dimensions

The Mohr's stress circle is used to find out graphically the direct stress σ and sheer stress_ on any plane inclined at θ to the plane on which $\sigma_x$acts.The direction of θ here is taken in anticlockwise dire ction from the BC.

STEPS:

In order to do achieve the desired o bjective we proceed in the following manner

(iv) Label the Block ABCD.

(v) Set up axes for the direct stress (as abscissa) and shear stress (as ordinate)

(vi) Plot the stresses on two adjace nt faces e.g. AB and BC, using the following signconvention.

Direct stresses_ tensile positive; compressive, negative Shear stresses 1 tending to turn block clockwise,

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positive1 tending to turn block counter clockwise,

negative

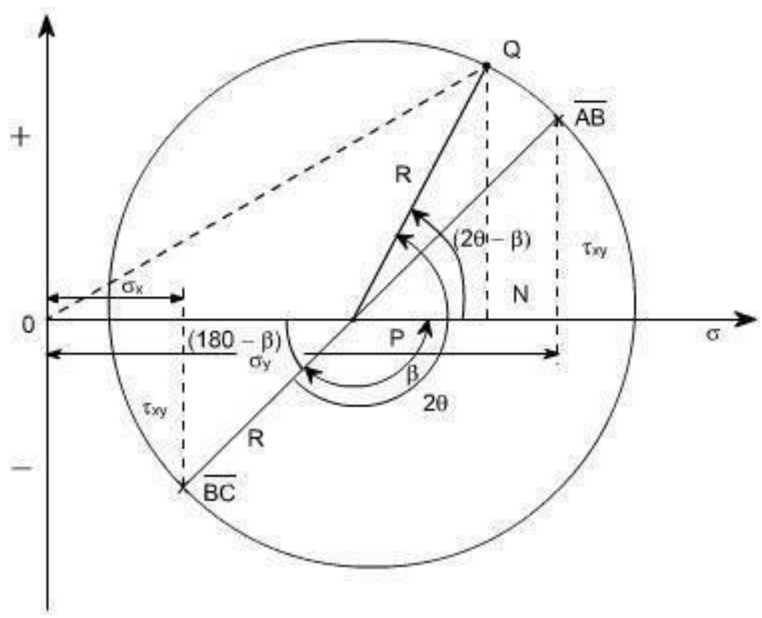
[i.e shearing stresses are +ve when its movement about the centre of the element is clockwise]

This gives two points on the graph which may than be labeled as respectively to denotestresses on these planes.

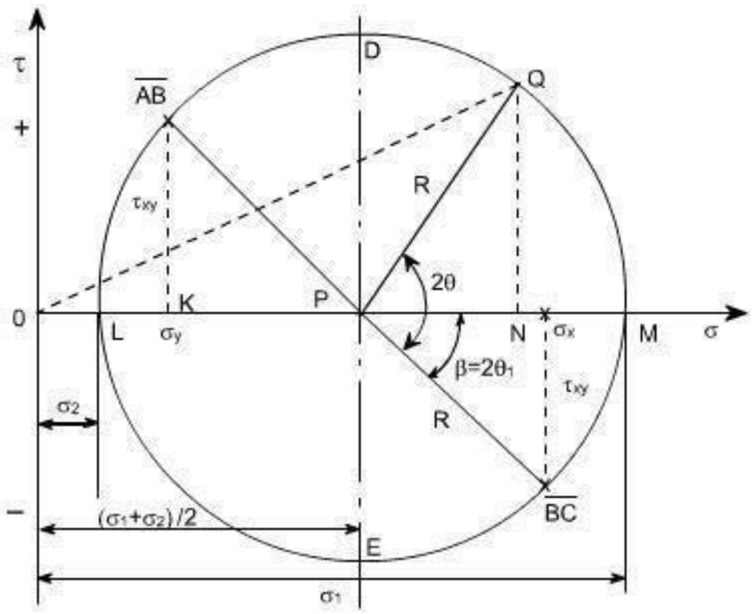
- Join .

- The point P where this line cuts the s axis is than the centre of Mohr's stress circle and the linejoining is diameter. Therefore the circle can now be drawn.

Now every point on the circle then r epresents a state of stress on some plane through C.



Proof:



Consider any point Q on the circum ference of the circle, such that PQ makes an angle 2 _with BC, and dropa perpendicular from Q to meet the s axis at N.Then OQ represents the resultant stress on the plane an

angle θ to BC. Here we have assu med that $\sigma_x > \sigma_y$

Now let us find out the coordinates of point Q. These are ON and

QN.From the figure drawn earlierON = OP + PNOP = OK + KP

$$OP = \sigma_y + 1/2\,(\sigma_x - \sigma_y)$$

$$= \sigma_y / 2 + \sigma_y / 2 + \sigma_x / 2 + \sigma_y / 2$$

$$= (\sigma_x + \sigma_y) / 2$$

PN = $R\cos(2\theta - \beta$

) hence ON = OP +

PN

$= (\sigma_x + \sigma_y) / 2 + R\cos(2\theta - \_)$

$= (\_ _x + \sigma_y) / 2 + R\cos 2\theta \cos\beta +$

$R\sin 2\theta \sin\beta$ now make the substitutions for

$R\cos\beta$ and $R\sin\beta$.

Thus,

ON = $1/2 (\_ _x + \sigma_y) + 1/2 (\_ _x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2$)

Similarly QM = $R\sin(2\theta - \beta)$

$= R\sin 2\theta \cos\beta - R\cos 2\theta \sin\beta$

Thus, substituting the values of R $\cos\beta$ and $R\sin\beta$, we get

QM = $1/2 (\sigma_x - \sigma_y)\sin 2\theta - \tau_{xy}\cos 2\theta$ (2)

If we examine the equation (1) and (2), we see that this is the same equation which we have already derivedanalytically

Thus the co-ordinates of Q are the normal and shear stresses on the plane inclined at θ to B C in the original stress system.

N.B: Since angle PQ is 2θ on Mohr's circle and not θ it becomes obvious that angles ar e doubled onMohr's circle. This is the only differ ence, however, as They are measured in the same directi on and from the same plane inbothfigures.Further points to be noted are :

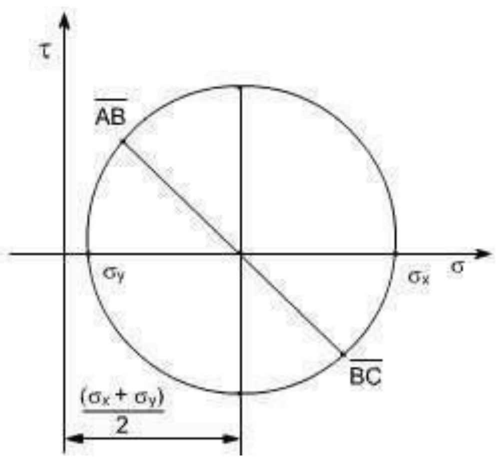
(1) The direct stress is maximum when Q is at M and at this point obviously the sheer stress is zero, hence by definition OM is the length representing the maximum principal stresses σ_1 and $2\theta_1$ gives the angle of theplane θ_1 from BC. Similar OL is the other principal stress and is represented by σ_2

(2) The maximum shear stress is given by the highest point on the circle and is represented y the radius ofthe circle.

This follows that since shear stresses and complimentary sheer stresses have the same value; therefore the centre of the circle will always lie o n the s axis midway between σ_x and σ_y. [since $+\tau_{xy}$ & $-\tau_{xy}$ are shearstress & complimentary shear stress so they are same in magnitude but different in sign.]

(3) From the above point the maxim um sheer stress i.e. the Radius of the Mohr's stress circle would be

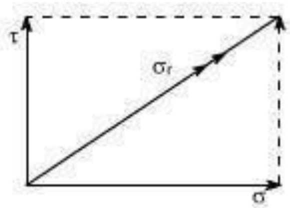
While the direct stress on the plane of maximum shear must be mid 1 may between σ_x and σ_y i.e



(4) As already defined the principal planes are the planes on which the shear components arezero.

Therefore are conclude that on principal plane the sheer stress is zero.

(5) Since the resultant of two stress at 90^0 can be found from the parallogram of vectors as shown in the diagram.Thus, the resultant stress on the plane at q to BC is given by OQ on Mohr's Circle.



(6) The graphical method of solution for a complex stress problems using Mohr's circle is a very powerful technique, since all the information relating to any plane within the stressed element is contained in the single construction. It thus, provides a convenient and rapid means of solution. Which is less prone to arithmetical errors and is highly recommended.

ILLUSRATIVE PROBLEMS:

Let us discuss few representative problems dealing with complex state of stress to be solvedeither analytically or graphically.

PROB 1: A circular bar 40 mm diameter carries an axial tensile load of 105 kN. What is the Value ofshear stress on the planes on which the normal stress has a value of 50 MN/m^2 tensile.

Solution:

Tensile stress $\sigma_y = F / A = 105 \times 10^3 / \pi \times (0.02)^2$

$= 83.55\ MN/m^2$

Now the normal stress on an obliqe plane is given by the relation

$\sigma_{\_\_} = \sigma_y \sin^2\theta$

$50 \times 10^6 = 83.55\ MN/m^2 \times 10^6 \sin^2\theta$

$\theta = 50^0 68'$

The shear stress on the oblique plane is then given by

$\tau_{\_} = 1/2\ \sigma_y \sin 2\theta$

$= 1/2 \times 83.55 \times 10^6 \times \sin 101.36$

$= 40.96\ MN/m^2$

Therefore the required shear stress is 40.96 MN/m^2

PROB 2:

For a given loading conditions the state of stress in the wall of a cylinder is expressed as follows:

(a) 85 MN/m^2 tensile

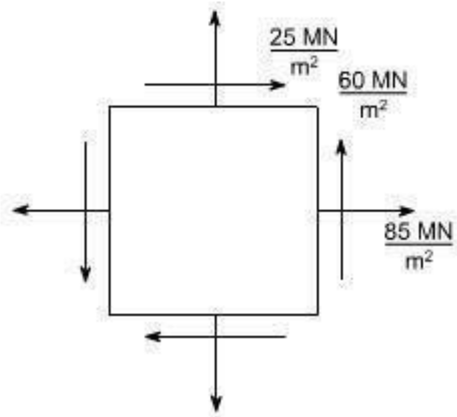
(b) 25 MN/m^2 tensile at right angles to (a)

(c) Shear stresses of 60 MN/m^2 on the planes on which the stresses (a) and (b) act; the sheer coupleacting on planes carrying the 25 MN/m^2 stress is clockwise ineffect.

Calculate the principal stresses and the planes on which they act. What would be the effect on these results if owing to a change of loading (a) becomes compressive while stresses (b) and (c) remain unchanged

Solution:

The problem may be attempted both analytically as well as graphically. Let us first obtain the analyticalsolution



The principle stresses are given by the formula

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σ_1 and σ_2

$$= \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}$$

$$= \frac{1}{2}(85 + 25) \pm \frac{1}{2}\sqrt{(85 + 25)^2 + (4 \times 60^2)}$$

$$= 55 \pm \frac{1}{2}.60\sqrt{5} = 55 \pm 67$$

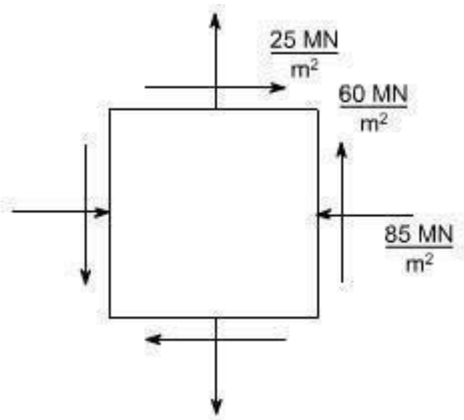
$$\Rightarrow \sigma_1 = 122 \text{ MN/m}^2$$

$$\sigma_2 = -12 \text{ MN/m}^2 \text{(compressive)}$$

For finding out the planes on which the principle stresses act us the equation

The solution of this equation will yeild two values θ i.e they θ_1 and θ_2 giving θ_1= $31^0 71'$ & θ_2= $121^0 71'$

(b) In this case only the loading (a) is changed i.e. its direction had been changed. While the otherstresses remains unchanged hence now the block diagram becomes.



Again the principal stresses would be given by the equation.

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$$\sigma_1 \;\&\; \sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$= \frac{1}{2}(-85 + 25) \pm \frac{1}{2}\sqrt{(-85 - 25)^2 + (4 \times 60^2)}$$

$$= \frac{1}{2}(-60) \pm \frac{1}{2}\sqrt{(-85 - 25)^2 + (4 \times 60^2)}$$

$$= -30 \pm \frac{1}{2}\sqrt{12100 + 14400}$$

$$= -30 \pm 81.4$$

$$\sigma_1 = 51.4 \text{ MN/m}^2;\ \sigma_2 = -111.4 \text{ MN/m}^2$$

Again for finding out the angles use the following equation.

$$\tan 2\theta = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

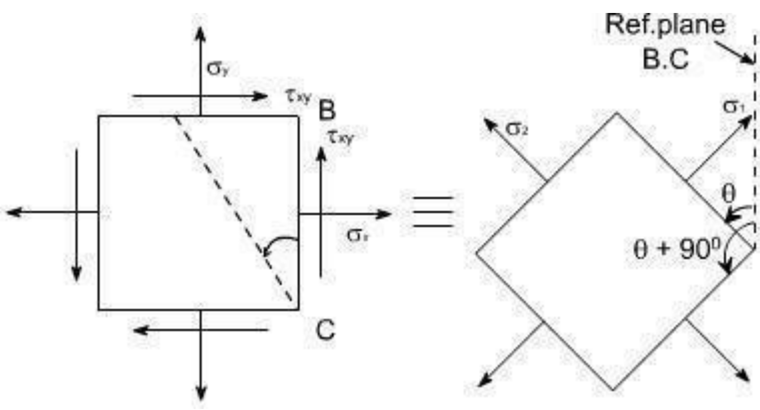
$$= \frac{2 \times 60}{-85 - 25} = +\frac{120}{-110}$$

$$= -\frac{12}{11}$$

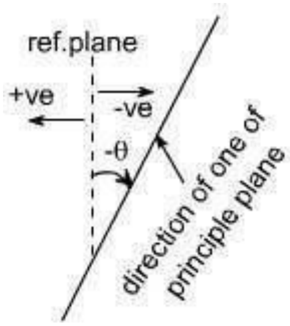
$$2\theta = \tan\left(-\frac{12}{11}\right)$$

$$\Rightarrow \theta = -23.74^0$$

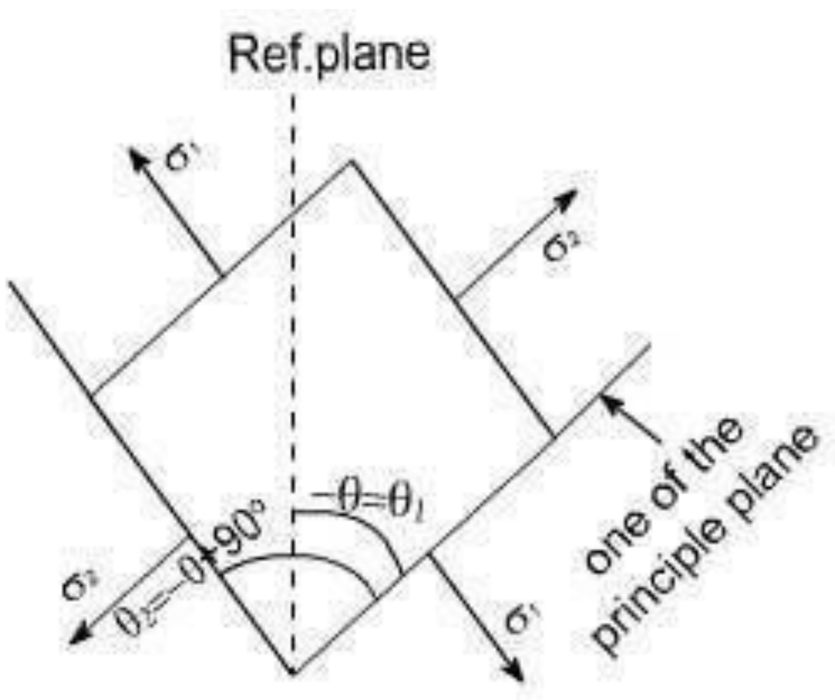
Thus, the two principle stresses acting on the two mutually perpendicular planes i.e principle planes may bedepicted on the element as shown below:



So this is the direction of one principle plane & the principle stresses acting on this would be σ_1 when is acting normal to this plane, now the direction of other principal plane would be $90^0 + \theta$ because the principal planes are the two mutually perpendicular plane, hence rotate the another plane $\theta + 90^0$ in the same direction to get the another plane, n ow complete the material element if θ is negative that means weare measuring the angles in the opposite direction to the reference plane BC



Therefore the direction of other principal planes would be $\{-\theta + 90\}$ since the angle $-\theta$ is always less inmagnitude then 90 hence the quantity $(-\theta + 90)$ would be positive therefore the Inclination of other plane with reference plane would be positive therefore if just complete the Block. It would appear as



If we just want to measure the angles from the reference plane, than rotate this block through 180^0 so asto have the following appearance.

So whenever one of the angles comes negative to get the positive value,

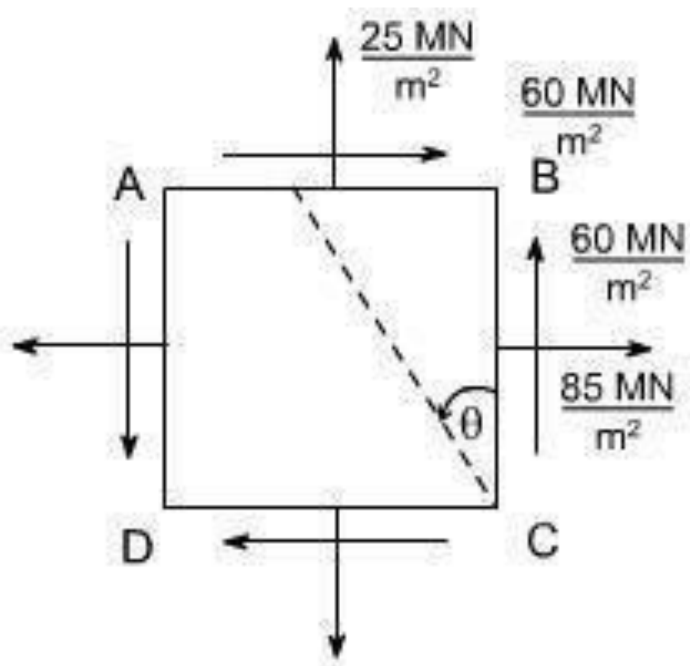
first Add 90^0 to the value and again add 90^0 as in this case $\theta = -23^0 74'$

so $\theta_1 = -23^0 74' + 90^0 = 66^0 26'$ .Again adding 90^0 also gives the direction of other

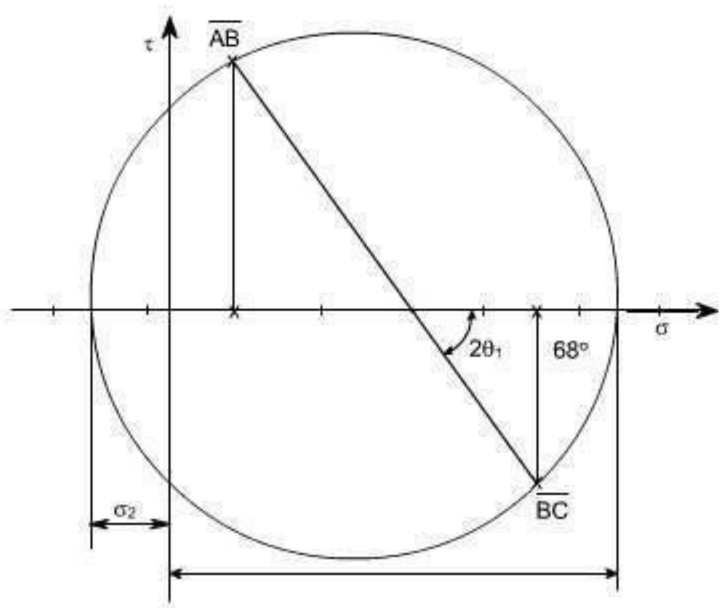
principle planesi.e $\theta_2 = 66^0 26' + 90^0 = 156^0 26'$
This is how we can show the angular position of these planes clearly.

GRAPHICAL SOLUTION:

Mohr's Circle solution: The same solution can be obtained using the graphical solution i.e the Mohr'sstress circle,for the first part, the block diagram becomes



Construct the graphical construction as per the steps given earlier.



Taking the measurements from the Mohr's stress circle, the various quantities computed are

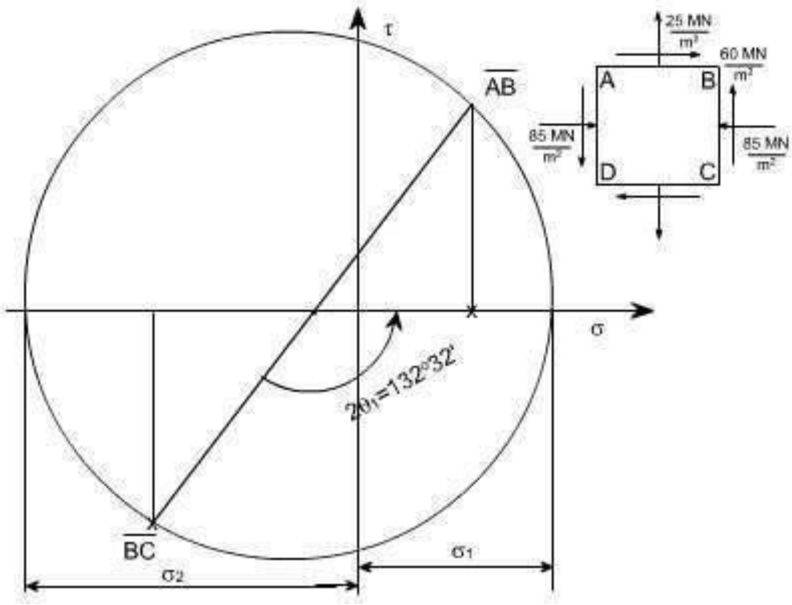
σ_1 = 120 MN/m^2 tensile

σ_2 = 10 MN/m^2 compressive

θ_1 = 34^0 counter clockwise from BC

θ_2 = 34^0 + 90 = 12^0 counter clockwise from

Part Second : The required configuration i.e the block diagram for this case is shown along with thestress circle.



By taking the measurements, the various quantites computed are given as

σ_1= 56.5 MN/m^2 tensile

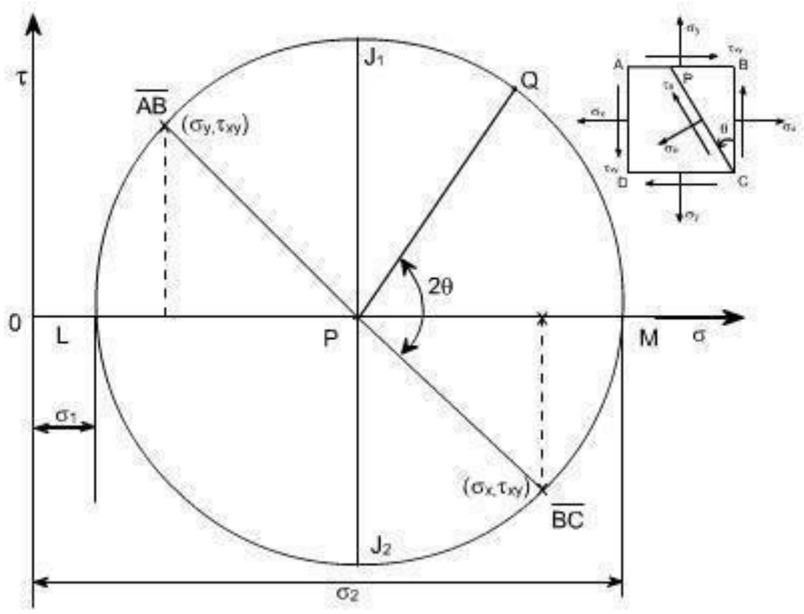
σ_2= 106 MN/m^2 compressive

θ_1= 66^0 15' counter clockwise from BC

θ_2= 156^0 15' counter clockwise from BC

Salient points of Mohr's stress circle:

1. complementary shear stresses (on planes 90^0 apart on the circle) are equal in magnitude
2. The principal planes are orthogonal: points L and M are 180^0 apart on the circle (90^0 apart in material)
3. There are no shear stresses on principal planes: point L and M lie on normal stress axis.
4. The planes of maximum shear are 45^0 from the principal points D and E are 90^0, measured roundthe circle from points L and M.
5. The maximum shear stresses are equal in magnitude and given by points D and E
6. The normal stresses on the planes of maximum shear stress are equal i.e. points D and E both havenormal stress co-ordinate which is equal to the two principalstresses.



As we know that the circle represen ts all possible states of normal and shear stress on any plane through a stresses point in a material. Further we have seen that the co-ordinates of the point 1Q' are seen to be the same as those derived from equilibrium of the element. i.e. the normal and shear stress com ponents on any plane passing through the point ca n be found using Mohr's circle. Worthy of note:

1. The sides AB and BC of the elem ent ABCD, which are 90^0 apart, are represented on the circleby and they are 180^0 apart.
2. It has been shown that Mohr's circle represents all possible states at a point. Thus, it can b e seen at a point. Thus, it, can be seen that two planes LP and PM, 180^0 apart on the diagram and therefore 90^0 apart

in the material, on which shear stress τ_θ is zero. These planes are termed as principal planes a nd normalstresses acting on them are known as principal

stresses.Thus , $\sigma_1 = OL$

$\sigma_2 = OM$

3. The maximum shear stress in an element is given by the top and bottom points of the circle i.e by points J_1 and J_2 ,Thus the maximum shear stress would be equal to the radius of i.e. $\tau_{max}= 1/2(\_\ _1 - \sigma_2)$,the corresponding normal stress is obviously the distance $_0$OP = $1/2\ (\_\ _x + \sigma_y)$, Further it can also be seen that the planes on which the shear stres s is maximum are situated 90 from the principal planes (on circle), and

45^0 in the material.

4. The minimum normal stress is jus t as important as the maximum. The algebraic minimum s tress couldhave a magnitude greater than that of the maximum principal stress if the state of stress wer e such that the centre of the circle is to the left of orgin.

i.e. if $\sigma_1 = 20$ MN/m^2 (say)

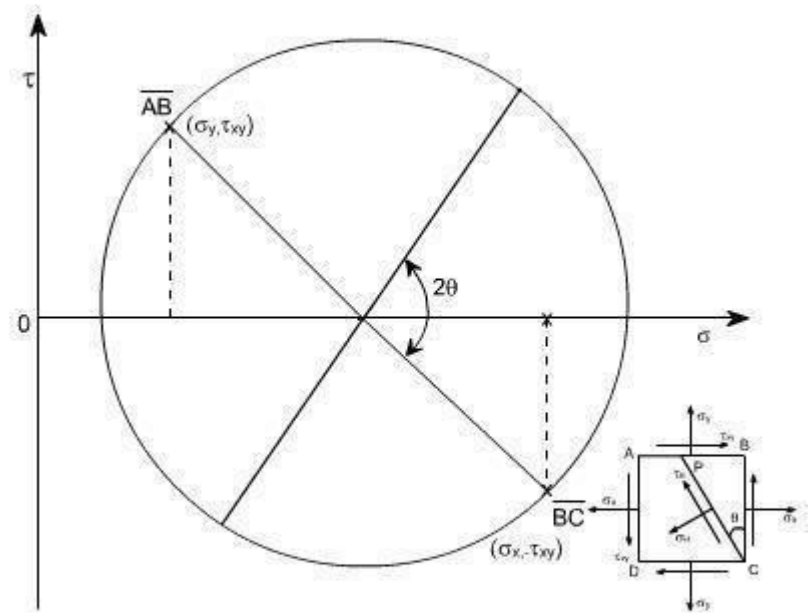
$\sigma_2 = -80$ MN/m^2 (say)

Then $\tau_{max}{}^{m} = (\sigma_1 - \sigma_2 / 2) = 50\ MN/m^2$

If should be noted that the principal stresses are considered a maximum or minimum mathem atically e.g.a compressive or negative stress is l ess than a positive stress, irrespective or numerical value.

5. Since the stresses on perpendul ar faces of any element are given by the co-ordinates of t wo diametrically opposite points on the circle, thus, the sum of the two normal stresses for any and all orientations of the element is constant, i.e. Thus sum is an invariant for any particular state of stress.

Sum of the two normal stress comp onents acting on mutually perpendicular planes at a point in a stateof plane stress is not affected by the o rientation of these planes.



This can be also understand from t he circle Since AB and BC are diametrically opposite thus, what ever may be their orientation, they will always lie on the diametre or we can say that their sum won't change, itcan also be seen from analytical relatio ns

We know on plane BC; $\theta = 0$ $\sigma_{n1} = \sigma_x$ on

plane AB; $\theta = 270^0$

$\sigma_{n2} = \sigma_y$

Thus $\sigma_{n1} + \sigma_{n2} = \sigma_x + \sigma_y$

6. If $\sigma_1 = \sigma_2$, the Mohr's stress circle degenerates into a point and no shearing stresses are d eveloped onxy plane.

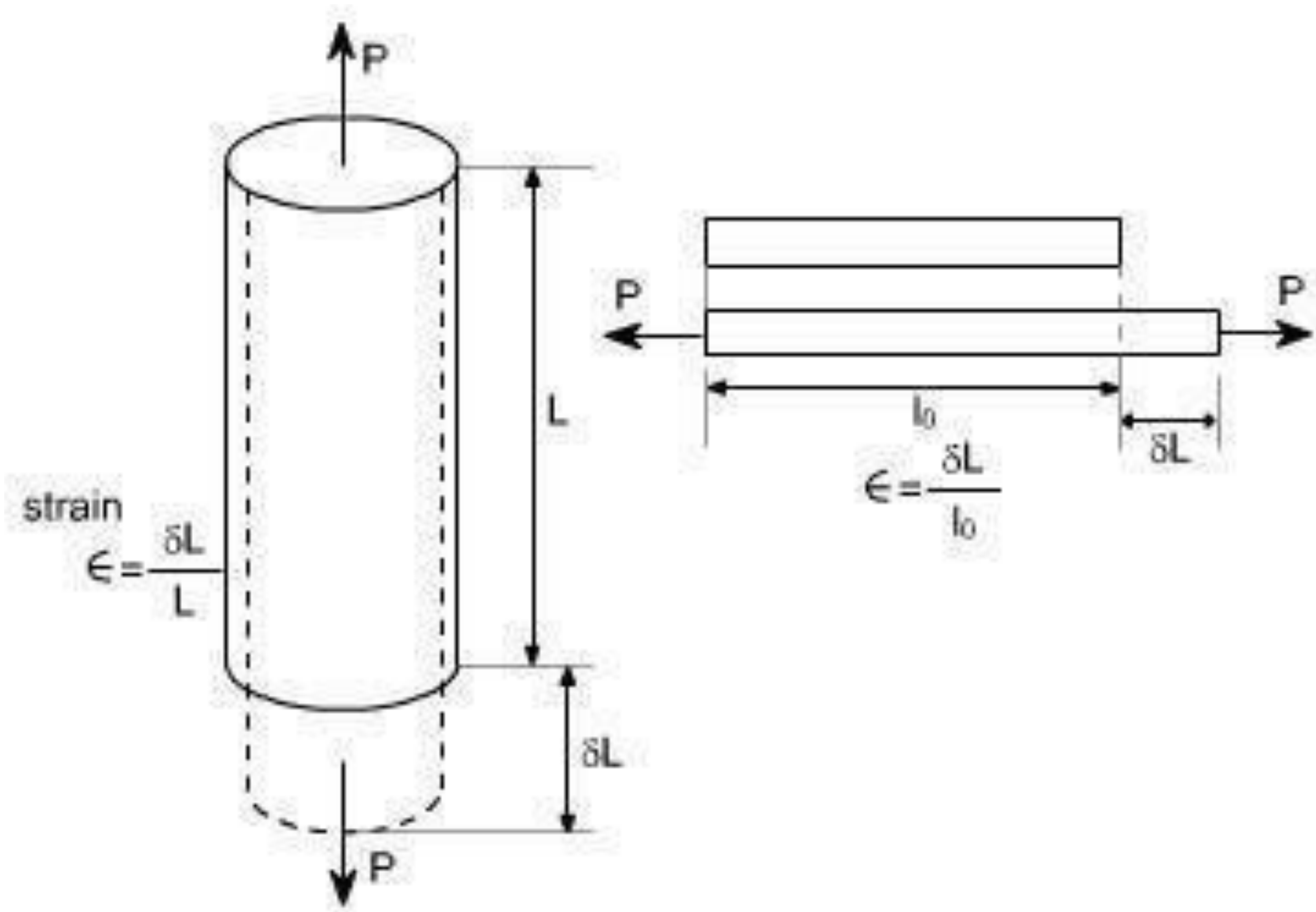
7. If $\sigma_x + \sigma_y = 0$, then the center of Mohr's circle coincides with the origin of $\sigma - \tau$ co-ordinates.

ANALYSIS OF STRAINS

CONCEPT OF STRAIN

Concept of strain : if a bar is subj ected to a direct load, and hence a stress the bar will change in length.If the bar has an original length L and changes by an amount δL, the strain produce is defined as follows:

Strain is thus, a measure of the deformation of the material and is a nondimensional Quantity i.e. it has nounits. It is simply a ratio of two qua ntities with the same unit.



Since in practice, the extensions of materials under load are very very small, it is often convenient to measurethe strain in the form of strain x 10^{-6} i.e. micro strain, when the symbol used becom es $\propto \in$.

Sign convention for strain:

Tensile strains are positive wherea s compressive strains are negative. The strain defined earlier wasknown as linear strain or normal strain or the longitudinal strain now let us define the shear strain.